
Agenda Item:	4.2.4
Source:	Ericsson, ST-Ericsson
Title:	Phase imbalance model and its impact to CSI reporting
Document for:	Discussion

1 Introduction

In previous meetings RAN 4 decided to progress the work on eDLMIMO CSI reporting requirements by considering the following:

RAN4 only defines the performance requirement without phase error impact and provides information on phase error impact on CQI and PMI performance to RAN5. RAN5 should discuss the margin. If margin is too much, then RAN4 can consider revisiting the test.

In the past meeting the discussion on the model for phase mismatch between the physical tx antennas started. It was shown in [1] that depending on the model and on the initial phase error the phase mismatch between the antennas can have high effect on the CSI reporting capability of the UE. This is more visible for static CQI tests where the channel is particularly matched to the precoding used for the transmission. Hence it is important to define a proper model for the phase mismatch which can mimic the behavior of realistic phase errors between the antenna branches. In general 2 possibilities were considered when starting the discussion on phase mismatch [2]:

- Case 1 the signal generator and the channel emulator are working in baseband. The RF conversion is done on the physical rx antennas. Hence only phase mismatch between the rx ports is assumed.
- Case 2 the signal generator generates the signal in baseband but it converts to RF level before sending them to the channel emulator. Possible phase mismatch arises between the physical tx antennas.

The assumption RAN 4 has considered so far is that Case 1 was applicable and hence no phase mismatch between the physical tx antennas was included for the definition of the requirements. Recently RAN 5 has sent back an LS to RAN 4 asking feedbacks on the required stability of the phase mismatch. Hence RAN 4 has to define a model and to study the impact of the possible phase mismatch into CSI requirements in order to provide feedbacks to RAN 5. This document addresses this issue.

2 Background

Lets consider the transmitted signal in time domain which is affected by the phase error on each antenna. Lets call $S_{k,\ell}$ the signal transmitted on subcarrier k and OFDM symbol ℓ , $\theta_p(t)$ the phase on antenna p which varies in time. The transmitted signal at time instant t on antenna port p can be written as

$$x_{\theta_p}(t) = \sum_{k=-N/2}^{N/2-1} S_{p,k,\ell} \exp \left[j2\pi k \Delta f \frac{t}{N} \right] e^{j\theta_p(t)} \quad (1)$$

where N is the number of subcarriers, $S_{p,k,\ell}$ is the constellation symbol sent on the k^{th} subcarrier for the ℓ^{th} OFDM symbol and n represents the component within the ℓ^{th} OFDM symbol, $n \in [0, N - 1]$. Note that the index p includes the dependency on the precoder.

Note that without loss of generality the index ℓ is omitted. If we now consider the signal received on a receiver port r transmitted by the P physical tx antennas $p = 0, \dots, P - 1$, it can be shown that the received signal after FFT at the receiver, at subcarrier m , can be written as (under AWGN channel)

$$z_m^r = \sum_{p=0}^{P-1} S_{p,m} C_p(0) + \sum_{p=0}^{P-1} \sum_{k=-N/2, k \neq m}^{N/2-1} S_{p,k} C_p(k - m) \quad (2)$$

where $C_p(k) = (1/N) \sum_{n=-N/2}^{N/2-1} e^{j2\pi \Delta f n k / N + j\theta_p(n)}$, as in [3]. This shows that there are two components for the UE. One component is constant w.r.t the subcarriers, and the second component is introducing a dependency with other subcarriers, hence introducing ICI. To avoid ICI the phase mismatch process should be constant. When considering CSI reporting, one could think that channel estimation could partially take account the effect of the phase error term. However due to the particular open loop CSI reporting test the chosen precoder does not match anymore the channel conditions and hence there will be a performance degradation or eventually the UE wont be able to pass the CSI test not because of poor UE implementation but because too high mismatch between the test conditions and the real conditions (considering phase mismatch) which the UE under test experiences. Hence it is important to

- Define a model which mimics this effect
- Define the boundaries of this mismatch
- Analyse the effect to decide whether particular tolerances should be accounted for in RAN 4 (through modification of the requirements) or RAN 5.

However it should be noted that depending on the implemented system (Case 1 or Case 2) the additional tolerances introduced to allow for a certain amount of phase mismatch between the physical antenna ports may allow for looser performance. Hence we think that the boundaries of the mismatch needs to be carefully addressed.

In literature it is shown that several models can be used in order to define the phase on each antenna. In this document we provide simulation results by considering a simplistic model described in Section 3 and previously proposed in [1], while we hint other possible more complex model in Appendix A.

3 Simplified phase error model

A phase error on each tx antenna $p = 0, \dots, P - 1$ is defined as follows

$$\theta_{p,n} = \epsilon_p \quad (3)$$

where $\epsilon_p \sim U(-\epsilon_{MAX}, \epsilon_{MAX})$ follows a Uniform distribution centered in 0 and it is constant for the duration of the simulation. The phase error between any couple of transmit antennas will be defined as

$$\Delta\theta_{p,k}^n = \theta_{p,n} - \theta_{k,n} = \epsilon_p - \epsilon_k \quad (4)$$

The pdf of this new random variable is given by the convolution between the pdf of the phase error on each antenna, i.e.

$$f_{\Delta\theta_{p,k}^n}(i) = \int f_{\theta_{p,n}}(n)f_{\theta_{p,n}}(i+n)dn = \frac{1}{2\epsilon_{MAX}}Tri(-2\epsilon_{MAX}, 2\epsilon_{MAX})$$

where $Tri(a, b)$ indicate the triangular function with support (a, b) . Hence all the phase errors between each couple of antennas follows the same distribution.

The simulations can be run as a function of a single parameter ϵ_{MAX} . In order to define the tolerances T , a post processing is needed according to the following formula:

$$T \text{ such that } \int_{-T}^T f_{\Delta\theta_{p,k}^n}(i)di = V \quad (5)$$

where 'V' is the requested level for the tolerance (e.g. 95%). By solving the above mentioned equation we can find the following 2 possible solutions of the integral above:

$$\begin{aligned} T^2 - 4\epsilon_{MAX}T + 4\epsilon_{MAX}^2V &= 0 \\ \rightarrow T_1 &= 2\epsilon_{MAX} \left(1 + \sqrt{1 - V}\right), T_2 = 2\epsilon_{MAX} \left(1 - \sqrt{1 - V}\right) \end{aligned} \quad (6)$$

where clearly the only valid solution is T_2 . Hence, the proposal is as follows:

Proposal: Define the phase error between any couple of antenna branches to be within $(-T_2, T_2)$, V per cent of the time, where T_2 is computed as $T_2 = 2\epsilon_{MAX} (1 - \sqrt{1 - V})$.

4 Impact on CQI

In order to see the impact of the tx phase mismatch between the branches we considered the test in Section 9.2.3.1 (AWGN, PUCCH 1-1) with the above mentioned model. 100 different random phases were considered. $\epsilon_{MAX} = \pm 10, \pm 20, \pm 30$ degree. For 10 and 20 degree all CQI tests passed the requirements for all SNR points; At $\epsilon_{MAX} = \pm 30$ degree a certain sensitivity of the test was experienced.

Tables 1, 2 and 3 provides the results in terms of number of median CQI vs SNR for 10, 20 and 30 deg phase error.

Table 1: Median CQI vs SNR for $\epsilon_{MAX} \pm 10$ deg.

SNR	Median CQI CW1				% reported CQI within medCQI ± 1
	8	9	11	12	
7	100	0	0	0	100
8	0	100	0	0	100
13	0	0	100	0	100
14	0	0	0	100	100
SNR	Median CQI CW2				% reported CQI within medCQI ± 1
	8	9	11	12	
7	100	0	0	0	100
8	0	100	0	0	100
13	0	0	100	0	100
14	0	0	0	100	100

Figures 1-4 show the CDF of the BLER at different SNR values (7, 8, 13 and 14dB) for follow CQI for different phase errors. It can be clearly see that increasing the phase error it increases the spread in BLER.

Even tough the CQI results show that in principle the UE could still pass the requirements as already defined in 36.101 even in presence of 30deg phase error, however it can be seen that the spread in terms of BLER becomes very large and the CQI test fails for one of the 2 SNR points. Hence, in order to make sure that there is negligible impact in the CQI static test, we propose to limit ϵ_{MAX} to 20deg. More simulations should be performed in order to check the behaviour of other CSI tests (CQI PMI and RI), even tough it was already recognized that the test of CQI in static condition is the most sensitive to those type of errors. Hence

Table 2: Median CQI vs SNR for $\epsilon_{MAX} \pm 20\text{deg}$.

SNR	Median CQI CW1				% reported CQI within medCQI ± 1
	8	9	11	12	
7	100	0	0	0	100
8	0	100	0	0	100
13	0	0	100	0	100
14	0	0	0	100	100
SNR	Median CQI CW2				% reported CQI within medCQI ± 1
	8	9	11	12	
7	100	0	0	0	100
8	1	99	0	0	100
13	0	0	100	0	100
14	0	0	0	100	100

Table 3: Median CQI vs SNR for $\epsilon_{MAX} \pm 30\text{deg}$.

SNR	Median CQI CW1				% reported CQI within medCQI ± 1
	8	9	11	12	
7	100	0	0	0	100
8	25	75	0	0	100
13	0	0	100	0	100
14	0	0	2	98	100
SNR	Median CQI CW2				% reported CQI within medCQI ± 1
	8	9	11	12	
7	100	0	0	0	100
8	25	75	0	0	100
13	0	0	100	0	100
14	0	0	2	98	100

the following proposals:

Proposal: Consider a Uniform distribution for the initial phase on each antenna branch (note this is the phase on each antenna branch and not the phase error between couple of antennas). Limit ϵ_{MAX} to 20deg. Run further simulations by checking the impact of other CSI tests with $\epsilon_{MAX} = 20\text{deg}$.

Additionally Figures 5-8 show the standard deviation of the reported CQI for the same SNR values. Again it can be seen that the variance of the reported CQI varies considerably when increasing the phase mismatch to 30deg. We think it is important to have BLER with low variation which means consistent CQI reporting.

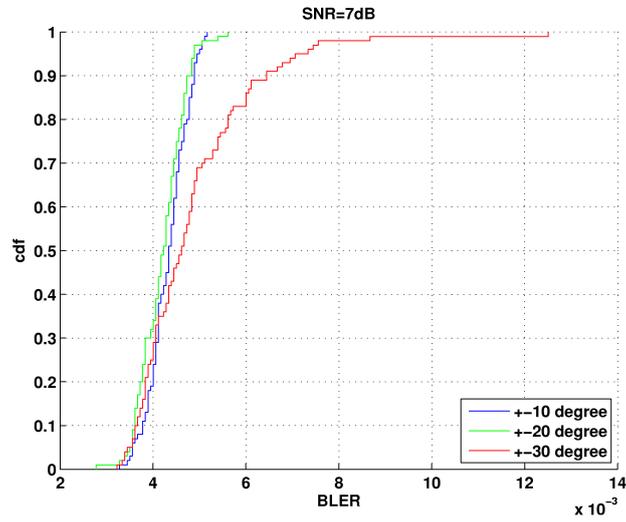


Figure 1: BLER at SNR 7dB for follow CQI for different phase errors.

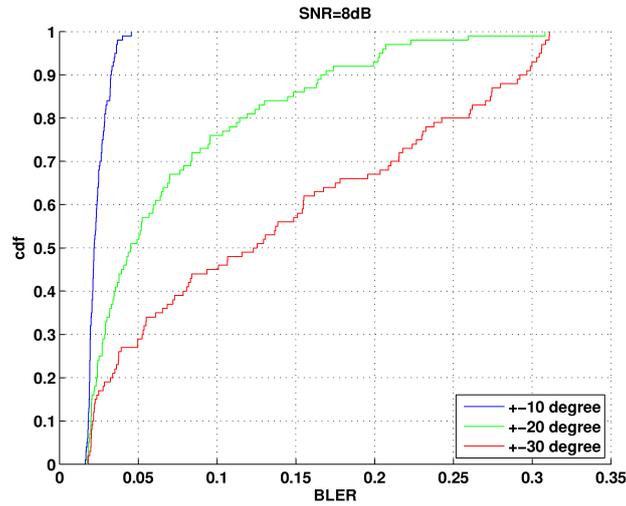


Figure 2: BLER at SNR 8dB for follow CQI for different phase errors.

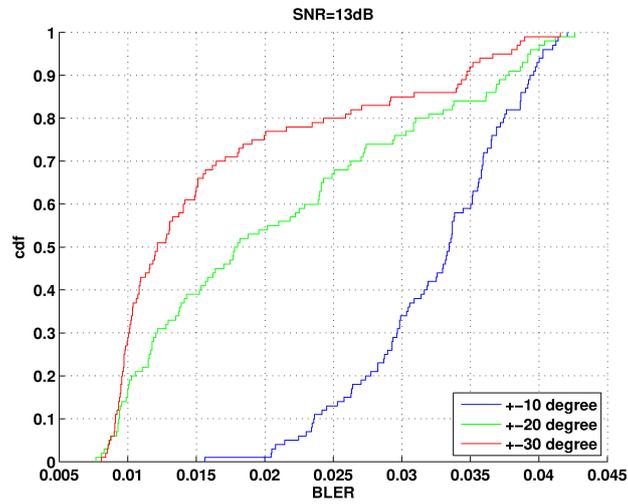


Figure 3: BLER at SNR 13dB for follow CQI for different phase errors.

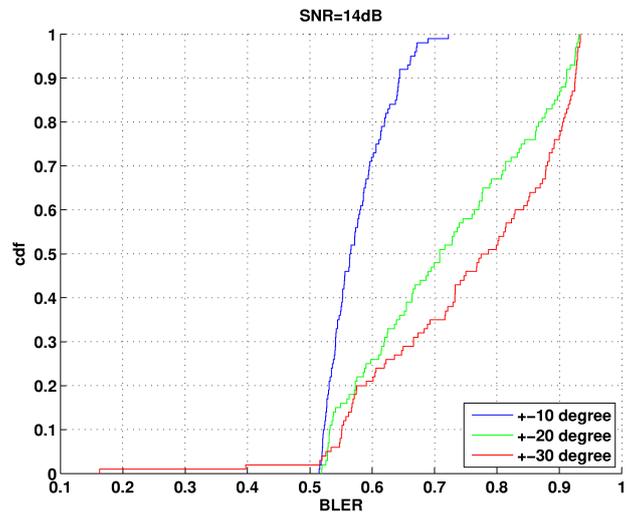


Figure 4: BLER at SNR 14dB for follow CQI for different phase errors.

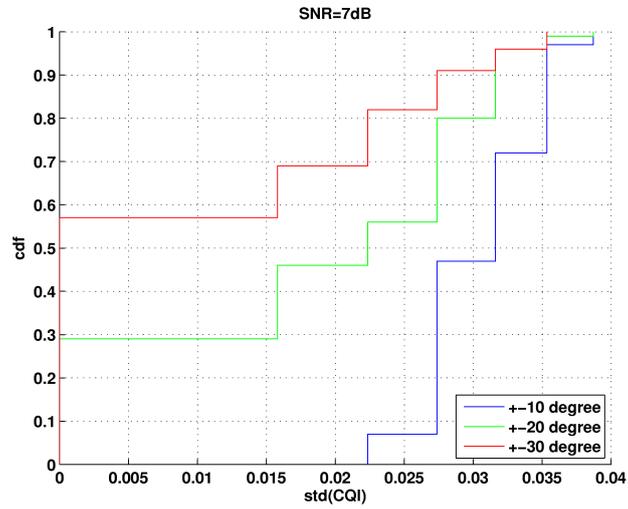


Figure 5: Standard deviation of the reported CQI at SNR 7dB for different phase errors.

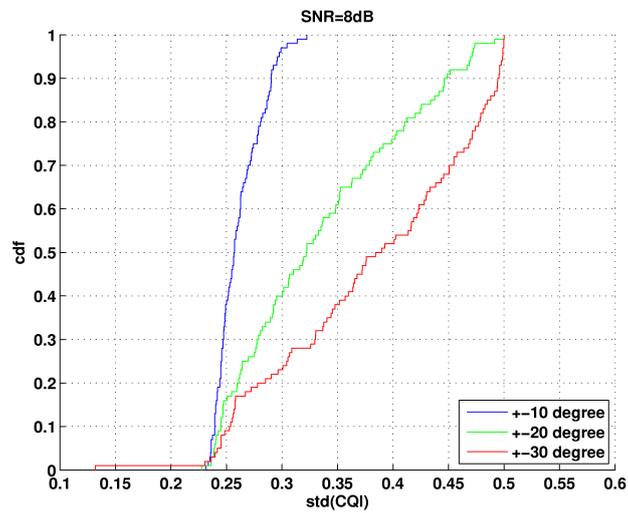


Figure 6: Standard deviation of the reported CQI at SNR 8dB for different phase errors.

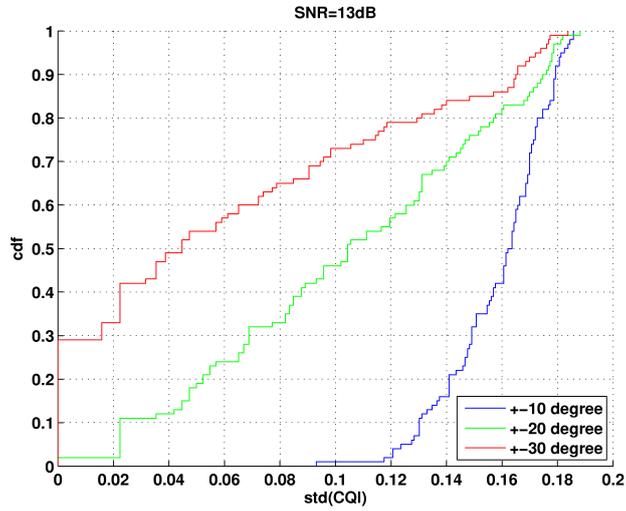


Figure 7: Standard deviation of the reported CQI at SNR 13dB for different phase errors.

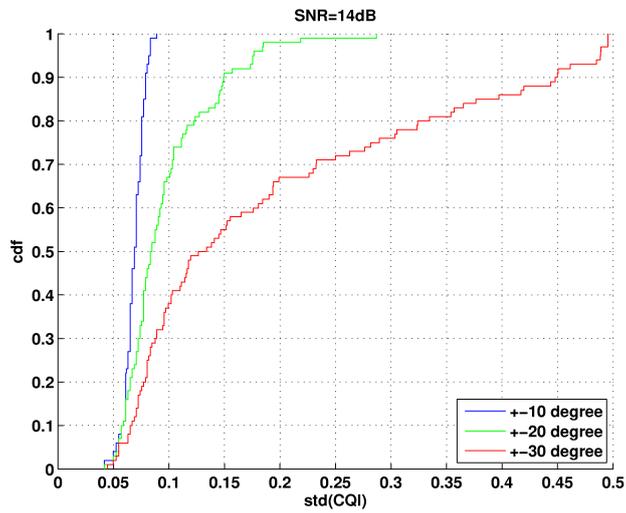


Figure 8: Standard deviation of the reported CQI at SNR 14dB for different phase errors.

5 Conclusions

In this document we have discussed the impact of the tx branches phase mismatch on CQI static performance. In particular we have considered a simplified model based on a static phase error on each tx branch with a Uniform distribution. Moreover we have hinted other possible more complex model in the Appendix. The following has been proposed:

Proposal: Define the phase error between any couple of antenna branches to be within $(-T_2, T_2)$ V per cent of the time, where T_2 is computed as $T_2 = 2\epsilon_{MAX} (1 - \sqrt{1 - V})$.

Proposal: Consider a Uniform distribution for the initial phase on each antenna branch (note this is the phase on each antenna branch and not the phase error between couple of antennas). Limit ϵ_{MAX} to 20deg. Run further simulations by checking the impact of other CSI tests with $\epsilon_{MAX} = 20\text{deg}$.

A Appendix: Other phase error model

Three cases are considered here:

- Case a. Brownian model
- Case b. Autoregressive model
- Case c. Moving average model

In the following we are going to define mean variance and time correlation for the three above mentioned cases.

A.1 Brownian Model

The sampled Brownian model can be written as

$$\theta_{BR,p,n} = \sum_{i=0}^n \epsilon_{i,p} \quad (7)$$

where $\epsilon_{i,p} \sim N(\Phi_p, \sigma_{\epsilon_p}^2)$, i.i.d random variable.

The mean is given by

$$E[\theta_{BR,p,n}] = (n + 1)\Phi_p \quad (8)$$

The variance is obtained as

$$Var[\theta_{BR,p,n}] = (n+1)Var(\epsilon_{i,p}) = (n+1)\sigma_{\epsilon_p}^2 \quad (9)$$

The time-autocorrelation given by:

$$\begin{aligned} R_{BR,\theta}^p(n,r) &= E[\theta_{p,n}\theta_{p,r}^*] = E\left[\sum_{i=0}^n \epsilon_{i,p} \sum_{j=0}^r \epsilon_{j,p}^*\right] \\ &= E\left[\sum_{i=0}^n \epsilon_{i,p}^2 + \sum_{i=0}^n \sum_{j=0, j \neq i}^n \epsilon_{i,p} \epsilon_{j,p}^* + \sum_{i=0}^n \sum_{j=n}^r \epsilon_{i,p} \epsilon_{j,p}^*\right] \\ &= E\left[\sum_{i=0}^{n-1} \epsilon_{i,p}^2\right] = \sum_{i=0}^{n-1} \sigma_{\epsilon_p}^2 + \Phi_p = (n+1)(\sigma_{\epsilon_p}^2 + \Phi_p) \end{aligned} \quad (10)$$

In autocorrelation in space domain is considered to be 0 as the processes are independent w.r.t index p .

A.2 Autoregressive model

The Autoregressive model can be written as

$$\theta_{AR,p,n} = \alpha\theta_{AR,p,n-1} + \epsilon_{n,p} = \sum_{i=0}^n \alpha^{n-i} \epsilon_{i,p} \quad (11)$$

where $\epsilon_{n,p} \sim N(\Phi_p, \sigma_{\epsilon_p}^2)$, i.i.d random variable.

The mean is given by

$$E[\theta_{AR,p,n}] = \Phi_p \sum_{i=0}^n \alpha^{n-i} = \Phi_p \frac{(\alpha^{n+1} - 1)}{(\alpha - 1)} \quad (12)$$

The variance is obtained as

$$Var[\theta_{AR,p,n}] = \sigma_{\epsilon_p}^2 \frac{(\alpha^{2n+2} - 1)}{(\alpha^2 - 1)} \quad (13)$$

The time-autocorrelation given by:

$$R_{BR,\theta}^p(n, n+L) = E[\theta_{p,n}\theta_{p,n+L}^*] = (\sigma_{\epsilon_p}^2 + \Phi_p)\alpha^L \frac{(\alpha^{2n+2} - 1)}{(\alpha^2 - 1)} \quad (14)$$

A.3 Moving average model

The Moving Average model can be written as

$$\theta_{MA,p,n} = \sum_{i=0}^n \gamma_i \epsilon_{n-i,p} \quad (15)$$

where $\epsilon_{n,p} \sim N(\Phi_p, \sigma_{\epsilon_p}^2)$, i.i.d random variable.

The mean is given by

$$E[\theta_{MA,p,n}] = \Phi_p \sum_{i=0}^n \gamma_i \quad (16)$$

The variance is obtained as

$$Var[\theta_{MA,p,n}] = \sigma_{\epsilon_p}^2 \sum_{i=0}^n \gamma_i^2 \quad (17)$$

The time-autocorrelation given by:

$$R_{BR,\theta}^p(n, n+L) = E[\theta_{p,n} \theta_{p,n+L}^*] = (\sigma_{\epsilon_p}^2 + \Phi_p) \sum_{i=0}^n \gamma_i \gamma_{i+L} \quad (18)$$

A.4 Selected Alternative model

In [4] and references therein it is shown that the single-sideband phase noise power follows a Lorentzian spectrum [4, 5] given by:

$$L(f) = \frac{2}{\pi \Delta f_{3dB}} \frac{1}{1 + [2f/(\Delta f_{3dB})]^2} \quad (19)$$

where Δf_{3dB} is the two-sided 3 dB bandwidth of phase noise; $\sigma_{\epsilon_p}^2 = 4\pi BT$ where $B = \Delta f_{3dB}/2$ and T is the sampling time.

However we think that as a first approximation the Gaussian hypothesis for the increment of the phase is a reasonable assumption. An alternative hypothesis would be to consider $\epsilon_{n,p} \sim N_{-\epsilon_{MAX}, \epsilon_{MAX}}(\Phi_p, \sigma_{\epsilon_p}^2)$, i.i.d random variable where $N_{a,b}(\mu, \sigma^2)$ represents the truncated normal distribution, i.e. $-\epsilon_{MAX} \leq \epsilon_{n,p} \leq \epsilon_{MAX}$.

The brownian model is not considered as suitable because of increasing mean and variance (one could consider normalization factors in order to maintain a certain mean and variance as a function of time). Moreover the autocorrelation does not depend on the time difference between instant n and r . The moving average model can be considered as more generic and gives more freedom in terms of the choice of the weighing factors.

However, for simplicity the autoregressive model with truncated gaussian random variable could be considered.

Note that the process is run independently on each tx antenna port and this will lead to a random mismatch between the physical tx antennas, however this is bounded as follows

$$\begin{aligned}
\Delta\theta_{p,q}^n &= \theta_{AR,p,n} - \theta_{AR,q,n} = \sum_{i=0}^n \alpha^{n-i} \epsilon_{i,p} - \sum_{i=0}^n \alpha^{n-i} \epsilon_{i,q} \\
&= \sum_{i=0}^n \alpha^{n-i} \delta \epsilon_{p,q}^i \leq 2\epsilon_{MAX} \sum_{i=0}^n \alpha^{n-i} \\
&= 2\epsilon_{MAX} \frac{1 - \alpha^{n+1}}{1 - \alpha} \tag{20}
\end{aligned}$$

which shows that $\Delta\theta_{p,q}^n \leq 2\epsilon_{MAX}$.

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