Title:	Modeling Intercell Interference
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1. INTRODUCTION

Intercell interference in MIMO downlink packet data systems caused by base station transmissions is spatially colored and is dependent on the MIMO channel realization between the transmitting base and the mobile receiver. Ideally, one would explicitly model the spatial characteristics of all interferering bases, however the complexity may be restrictive. Alternatively, one could model the spatial characteristics of the interferers whose powers exceed a given fraction of the serving base's power. The impact of this simplifying measure on the MMSE receiver structure was addressed in [1]. In this document, we extend the study and consider the impact on both the MMSE receiver and the SINR computation. We also consider spatially correlated channels.

2. METHODOLOGY

The MMSE receiver has been proposed as a baseline receiver for comparing system performance results [2]. The tap weights of the MMSE receiver are a function of the spatial characteristics of the interference from adjacent cells. One can account for the interference explicitly by the modeling it as spatially colored noise, or one can model it as spatially white noise. By modeling the interference as spatially white noise, the resulting MMSE receiver would not account for the spatial nature of the interference, and the SINR output may not accurately reflect the SINR output of an MMSE receiver that accounted for it. For example, if there is strong spatial interference from a given direction, the MMSE receiver that accounts for it may be able to suppress it and give a much higher SINR. On the other hand, by modeling the interference from all bases explicitly, while the MMSE receiver may more closely reflect an adaptive MMSE receiver that may be implemented in practice, the complexity in modeling the interference completely may be restrictively high. Using the MMSE the completely models the interference as a baseline, we use the system simulation methodology proposed in [3] and evaluate the error in the SINR at the output of the MMSE receiver as the amount of explicitly modeled interference is reduced.

We consider the received signal by a mobile receiver with R antennas in the center cell of a system with 2 rings of hexagonal cells, each with 3 sectors for a total of 57 sectors. Each of the bases has T transmit antennas. As in [3], the received powers from all 57 sectors are determined based on path loss and shadow fading, and the sector with the strongest power is chosen to be the serving sector. Let the received signal for a given chip interval be given by

$$\mathbf{r} = \sum_{i=0}^{56} \frac{A_i}{\sqrt{T}} \mathbf{H}_i \mathbf{b}_i \tag{1}$$

where A_i (i = 0, 1, …, 56) is the amplitude (per transmit antenna) of the signal from the ith sector, $\mathbf{H}_i$ is the R-by-T channel matrix corresponding to this sector, $\mathbf{b}_i$ is the T-dimensional vector corresponding to the chip elements of the T transmitted signals from this sector. For simplicity, we ignore the presence of additive white Gaussian noise. The elements of the matrices $\mathbf{H}_i$ are i.i.d., complex Gaussian random variables with unit power. In general, the elements of these matrices are derived from the spatial channel model. The elements of $\mathbf{b}_i$ have unit power, and the index i = 0 corresponds to the serving sector so that A_0 is greater than all other A_i (i = 1, 2,

…, 56). The other sectors can be ordered in an arbitrary order. Given the received signal, the MMSE receiver is the R-by-T matrix [2]:

$$\mathbf{W} = \frac{\sqrt{T}}{A_0}\mathbf{H}_0\left(\mathbf{H}_0^H\mathbf{H}_0 + \sum_{i=1}^{56}\frac{TA_i^2}{A_0^2}\mathbf{H}_i^H\mathbf{H}_i\right)^{-1} \tag{2}$$

Suppose we wish to spatially model the interference from only a subset of the bases. More specifically, let $F(w)$ denote this subset of sector indices such that $A_i / A_0 > w$. Therefore $F(0)$ is the entire index set (i = 1, 2, …, 56), and $F(1)$ is the null set. The covariance of the signal from the ith base is

$$E\left[\frac{A_i^2}{T}\mathbf{H}_i\mathbf{b}_i\mathbf{b}_i^H\mathbf{H}_i^H\right] = \frac{A_i^2}{T}E\left[\mathbf{H}_i\mathbf{H}_i^H\right] = \frac{A_i^2}{T}\begin{bmatrix} T & & \\ & \ddots & \\ & & T\end{bmatrix} = A_i^2\mathbf{I}_R \tag{3}$$

where $\mathbf{I}_R$ is the R-by-R identity matrix. Assuming that the interference from bases outside of set $F(w)$ can be modeled as spatially white Gaussian noise, the received signal in (1) can be rewritten as

$$\mathbf{r} = \frac{A_0}{\sqrt{T}}\mathbf{H}_0\mathbf{b}_0 + \sum_{i\in F(w)}\frac{A_i}{\sqrt{T}}\mathbf{H}_i\mathbf{b}_i + \sum_{i\notin F(w)}A_i\mathbf{n}_i \tag{4}$$

where $\mathbf{n}_i$ is a complex Gaussian random vector with zero mean and variance $\mathbf{I}_R$. Therefore the MMSE receiver which models the spatial interference from only the set $F(w)$ explicitly is

$$\mathbf{W}(w) = \frac{\sqrt{T}}{A_0}\mathbf{H}_0\left(\mathbf{H}_0^H\mathbf{H}_0 + \sum_{i\in F(w)}\frac{A_i^2}{A_0^2}\mathbf{H}_i^H\mathbf{H}_i + \sum_{i\notin F(w)}\frac{TA_i^2}{A_0^2}\mathbf{I}_T\right)^{-1}. \tag{5}$$

Note that for $w = 0$, $\mathbf{W}(w)$ in (2) is equivalent to $\mathbf{W}$ in (1). We write the matrices $\mathbf{H}_0$ and $\mathbf{W}(w)$ in terms of their column vectors $\mathbf{H}_0 = \left[\mathbf{h}_{0,1}\cdots\mathbf{h}_{0,T}\right]$ and $\mathbf{W}(w) = \left[\mathbf{w}_1(w)\cdots\mathbf{w}_T(w)\right]$, and we write the data vector $\mathbf{b}_0$ in terms of its components $\mathbf{b}_0 = [b_{0,1}\ldots b_{0,T}]^T$. The output of the MMSE receiver for the tth antenna (t = 1 … T) is the inner product between the column vector $\mathbf{w}_t(w)$ and the received signal $\mathbf{r}$ given by (1):

$$\mathbf{w}_t^H(w)\mathbf{r} = \frac{A_0}{\sqrt{T}}\mathbf{w}_t^H(w)\mathbf{h}_{0,t}b_{0,t} + \frac{A_0}{\sqrt{T}}\sum_{j=1, j\neq t}^{T}\mathbf{w}_t^H(w)\mathbf{h}_{0,j}b_{0,j} + \sum_{i=1}^{56}\frac{A_i}{\sqrt{T}}\mathbf{w}_t^H(w)\mathbf{H}_i\mathbf{b}_i \tag{6}$$

The SINR for the tth antenna (t = 1 … T) can be computed from (6):

$$SINR_{t,full}(w) = \frac{A_0^2\left|\mathbf{w}_t^H(w)\mathbf{h}_t\right|^2}{A_0^2\sum_{\substack{j=1\\ j\neq t}}^{T}\left|\mathbf{w}_t^H(w)\mathbf{h}_j\right|^2 + \sum_{i=1}^{56}A_i^2\left\|\mathbf{w}_t^H(w)\mathbf{H}_i\right\|^2}, \tag{7}$$

where the *full* subscript denotes the fact that the spatial interference was fully modeled in computing the SINR. The derivation of the MMSE receiver and SINR up to this point was already given in [1]. We now extend the evaluation of SINR to the case where the interference term in the denominator accounts for the spatial interference characteristics for only set $F(w)$. In other words, we write the out put of the MMSE receiver assuming that the received signal is given by (4):

$$\mathbf{w}_t^H(w)\mathbf{r} = \frac{A_0}{\sqrt{T}}\mathbf{w}_t^H(w)\mathbf{h}_{0,t}b_{0,t} + \frac{A_0}{\sqrt{T}}\sum_{j=1,j\neq t}^{T}\mathbf{w}_t^H(w)\mathbf{h}_{0,j}b_{0,j} + \sum_{i=1}^{56}\frac{A_i}{\sqrt{T}}\mathbf{w}_t^H(w)\mathbf{H}_i\mathbf{b}_i \tag{8}$$

The SINR for the tth antenna ($t = 1 \ldots T$) can be computed from (8):

$$SINR_{t\,approx}(w) = \frac{A_0^2\left|\mathbf{w}_t^H(w)\mathbf{h}_t\right|^2}{A_0^2\sum_{\substack{j=1\\ j\neq t}}^{T}\left|\mathbf{w}_t^H(w)\mathbf{h}_j\right|^2 + \sum_{i\in F}A_i^2\left\|\mathbf{w}_t^H(w)\mathbf{H}_i\right\|^2 + \sum_{i\notin F}A_i^2\left\|\mathbf{w}_t^H(w)\right\|^2}, \tag{9}$$

where *approx* denotes that the spatial interference is modeled approximately. In the next section, we refer to the SINR expressions in equations (7) and (9) as "full SINR" and "approximate SINR," respectively.

3. NUMERICAL RESULTS

We compute the SINR in equations (7) or (9) for N channel realizations, and we denote the nth realization (n = 1 … N) as $SINR_{t,full}(w,n)$ and $SINR_{t\,approx}(w,n)$. Recall that for w = 0, $SINR_{t,full}(w,n)$ and $SINR_{t\,approx}(w,n)$ are computed using the MMSE when all of the interference is accounted for. We compute N = 1000 realizations of $SINR_{t,full}(w,n)$ and $SINR_{t\,approx}(w,n)$ (w = 0, 0.01, 0.1, 1) by randomly placing a user uniformly in the center cell of a 19-cell, two-ring hexagonal cell configuration. We let

$$e_{t\,full}(w,n) := \left|\frac{SINR_{t\,full}(w,n) - SINR_{t\,full}(0,n)}{SINR_{t\,full}(0,n)}\right|$$

be the normalized error between $SINR_{t,full}(w,n)$ and $SINR_{t,full}(0,n)$ for the nth realization. Similarly, we let

$$e_{t,approx}(w,n) := \left|\frac{SINR_{t,approx}(w,n) - SINR_{t\,full}(0,n)}{SINR_{t,full}(0,n)}\right|$$

be the normalized error between $SINR_{t\,approx}(w,n)$ and $SINR_{t,full}(0,n)$ for the nth realization. Note that the error is always measured with respect to $SINR_{t,full}(0,n)$, the calculation of SINR using the full modeling of the spatial interference for both the MMSE receiver and the interference in the SINR.

We collect the errors for all $t = 1 \ldots T$ and $n = 1 \ldots N$ and plot its cumulative distribution function (CDF) in Figure 1 for spatially uncorrelated channels, and in Figure 2 for spatially correlated channels (with the channel parameters given in Table 1). Figures 3 and 4 give the CDF of the number of sectors whose interference levels are higher than w. In Figure 1, starting from the left, the CDF of the error for w = 0.01 is less than 1% about 75% of the time using the full SINR model in equation (7). Simplifying the SINR calculation using the approximate SINR in equation (9), the overall error increases and is less than 1% about 45% of the time. Figure 3 shows that for w = 0.01, the fraction of interference power versus the serving base power $\mathbf{A}_0$ from 11 out of 56 sectors is less than $w = 0.01$ for over 90% of the realizations. Therefore, by ignoring the spatial structure of a significant number of interferers, the inaccuracy in the SINR output is minimal for a significant fraction of time. One can simplify the channel model even more by increasing w and decreasing the number of explicitly modeled interferers. For correlated channels, both the error and the number of modeled interferers increases with respect to the uncorrelated channels for a given w, indicating that approximating a realization of the correlated interference as spatially white noise is not as accurate.

BS antenna separation	4 wavelengths
MS antenna separation	0.5 wavelengths
BS PAS, angle spread	Laplacian, 35 degrees
MS PAS, angle spread	Laplacian, 5 degrees

Table 1. Parameters of spatially correlated channels

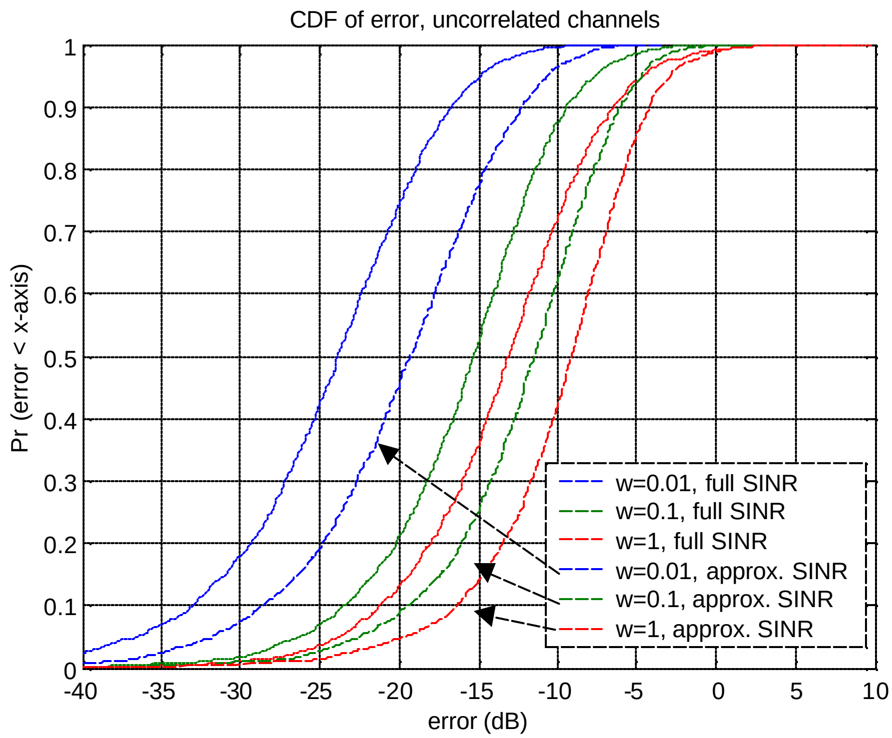

Figure 1. CDF of error, uncorrelated channels

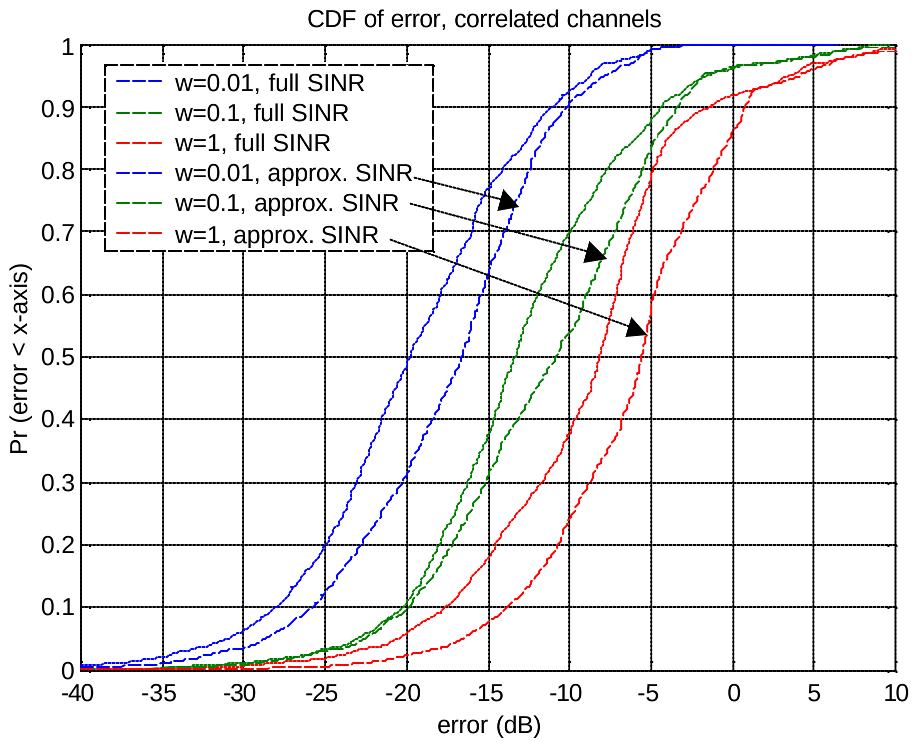

Figure 2. CDF of error, correlated channels

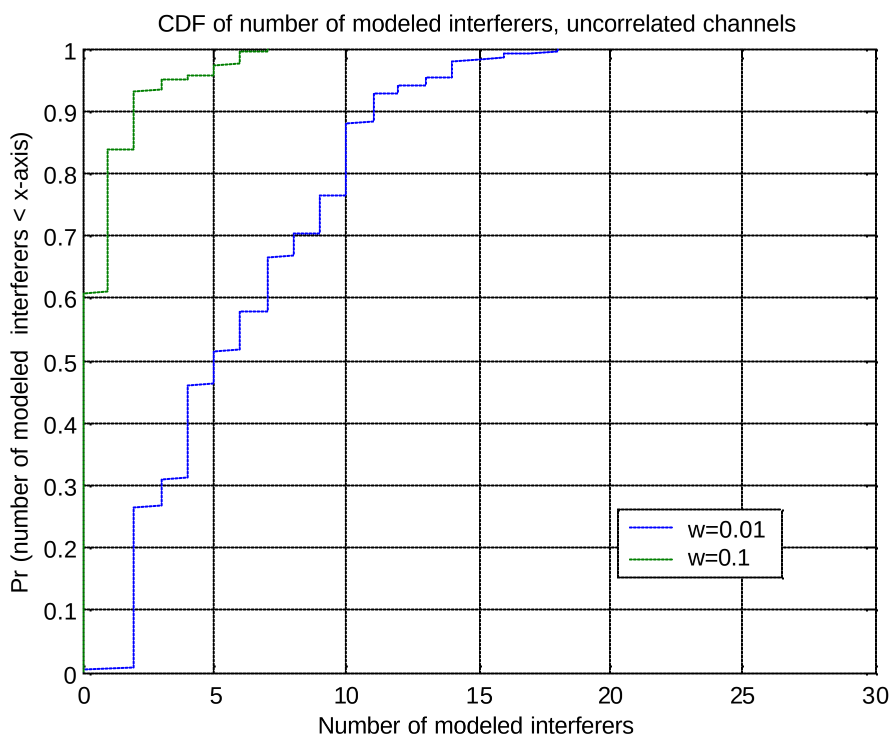

Figure 3. CDF of number of modeled interferes, uncorrelated channels

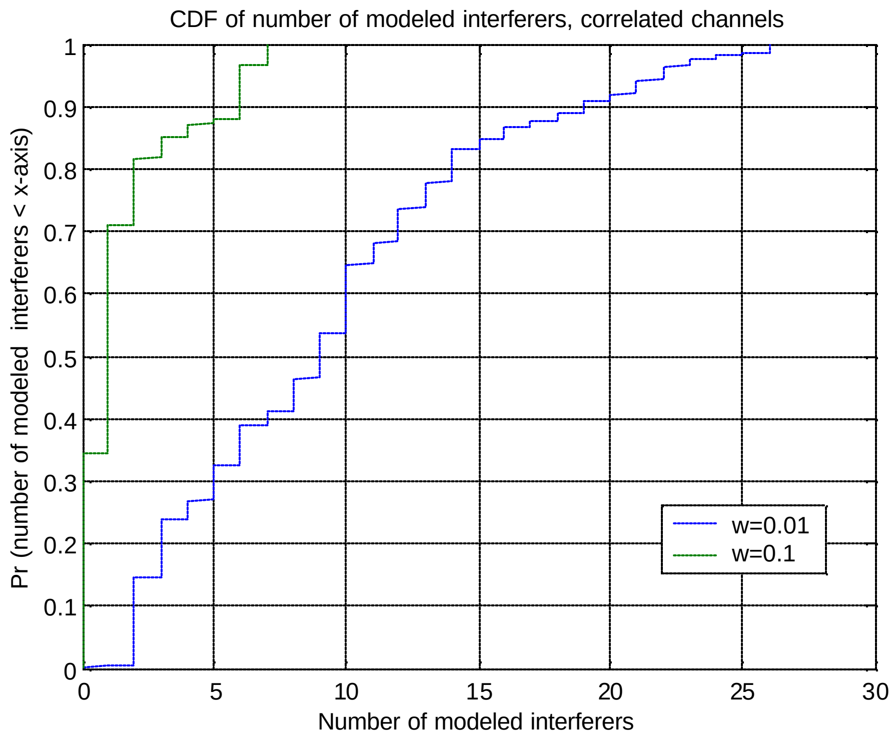

Figure 4. CDF of number of modeled interferes, correlated channels

4. CONCLUSIONS

We propose a threshold-based technique for determining how to account for intercell interference. We show that by modeling only a small fraction of the strongest interferers spatially, the system simulation complexity can be reduced significantly without significantly impacting the resulting SINR measurement at the output of an MMSE detector. Based on these observations, we propose that the system simulation methodology follow a similar technique to account for intercell interference.

5. REFERENCES

[1] SCM-071, Lucent, “Modeling intercell interference”

[2] SCM-056, Motorola, “A proposed receiver for system evaluations.”

[3] SCM-058, Lucent, “Preliminary system-level simulation results.”