TSG-RAN Working Group 1 meeting #6
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**TITLE:**

**Additional Results on Crosscorrelations of Differential Decoding for Current RACH Preambles**

**SOURCE:**

**Texas Instruments, Motorola**

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## 1.0 Introduction

In [1], Motorola and TI have shown the advantages of using the proposed RACH preambles in [1] over the currently accepted RACH preambles in namely (1) Improved cross-correlation properties (2) Flexibility in either coherent, non-coherent or differential decoding of the proposed preambles. In the report we show a problem with the currently accepted differentially encoded preambles. In all the analysis so far for the differentially encoded preambles, it has always been assumed that a single user is transmitting and the probability of error has been analyzed. However, in reality we could have several users transmitting simultaneously and hence differential decoding should also be analyzed in the presence of several users. As a simple case, we have assumed that there may be two simultaneous users with a zero lag transmitting two differentially encoded preambles. In this case, the cross correlation amongst the preambles due to differential decoding should also be analyzed. Table (1) below enumerates the cross correlation between the different users in dB below the main peak which is normalized to 0 dB. We can see from the table that for example, if the differentially encoded codes 1 and 10 are transmitted simultaneously then the cross correlation from code 10 to code 1 is only 4 dB below and from code 1 to code 10 is also only 10 dB below. This is illustrated in Fig. 1 where signature 10 is received with 4 dB more power than signature 1. After differential detection of signature 1, the output consists of a signal proportional to signature 1 plus cross terms which have the same power as signature 1. Because these terms have arbitrary relative phase, the detection performance can be severely degraded. A derivation of the cross terms is given in the Appendix. This will lead to a very high probability of miss for either the code 0 or the code 10 depending upon their relative received powers. Thus, we can see that the proposed differentially encodes may have very high cross correlation peaks when two or more codes are simultaneously transmitted with a zero lag leading to high probabilities of miss.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | -18 | -18 | -18 | -18 | -18 | -8 | -18 | -18 | -4 | -18 | -18 | -18 | -4 | -8 | -18 |
| 2 | -18 | | -4 | -18 | -18 | -18 | -18 | -4 | -8 | -18 | -18 | -8 | -18 | -18 | -18 | -18 |
| 3 | -18 | -4 | | -8 | -18 | -8 | -18 | -18 | -18 | -18 | -18 | -18 | -18 | -18 | -18 | -4 |
| 4 | -18 | -18 | -8 | | -18 | -18 | -18 | -8 | -4 | -18 | -18 | -4 | -18 | -18 | -18 | -18 |
| 5 | -18 | -18 | -18 | -18 | | -18 | -4 | -18 | -18 | -8 | -18 | -18 | -18 | -8 | -4 | 0 |
| 6 | -18 | -18 | -8 | -18 | -18 | | -18 | -8 | -4 | -18 | -18 | -4 | -18 | -18 | -18 | -18 |
| 7 | -8 | -18 | -18 | -18 | -4 | -18 | | -18 | -18 | -18 | -4 | -18 | -8 | -18 | -18 | -18 |
| 8 | -18 | -4 | -18 | -8 | -18 | -8 | -18 | | -18 | -18 | -18 | -18 | -18 | -18 | -18 | -4 |
| 9 | -18 | -8 | -18 | -4 | -18 | -4 | -18 | -18 | | -18 | -18 | -18 | -18 | -18 | -18 | -8 |
| 10 | -4 | -18 | -18 | -18 | -8 | -18 | -18 | -18 | -18 | | -8 | -18 | -4 | -18 | -18 | -18 |
| 11 | -18 | -18 | -18 | -18 | -18 | -18 | -4 | -18 | -18 | -8 | | -18 | -18 | -8 | -4 | -18 |
| 12 | -18 | -8 | -18 | -4 | -4 | -18 | -18 | -18 | -18 | -18 | -18 | | -18 | -18 | -18 | -8 |
| 13 | -18 | -18 | -18 | -18 | -18 | -18 | -8 | -18 | -18 | -4 | -18 | -18 | | -4 | -8 | -18 |
| 14 | -4 | -18 | -18 | -18 | -8 | -18 | -18 | -18 | -18 | -18 | -8 | -18 | -4 | | -18 | -18 |
| 15 | -8 | -18 | -18 | -18 | -4 | -18 | -18 | -18 | -18 | -18 | -4 | -18 | -8 | -18 | | -18 |
| 16 | -18 | -18 | -4 | -18 | -18 | -18 | -18 | -4 | -8 | -18 | -18 | -8 | -18 | -18 | -18 | |

**Table 1: The cross correlations amongst the differentially encoded codes is given when 2 codes are received simultaneously with zero lag between them. We can see for example that when codes 1 and 10 are received simultaneously, large cross correlations only 4 dB below the main peak occur.**

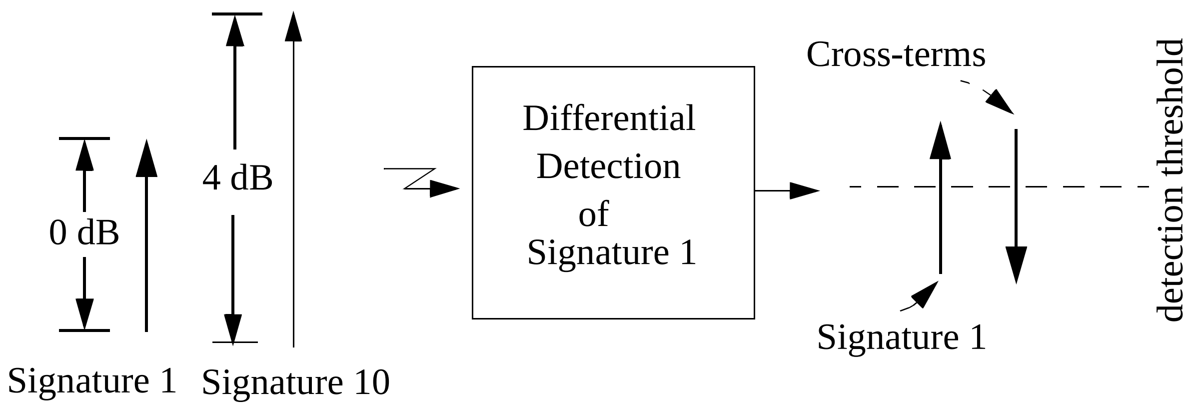


Figure 1: Result of differential detection when two signatures are present.

**Appendix: Derivation of Cross-terms**

The differentially encoded preamble signatures are obtained by differentially encoding a set of orthogonal preambles. Let $a_n$ and $\bar{a}_n$ be two orthogonal preambles which take values of $\pm 1$. The corresponding differentially encoded preambles are then defined by $b_n = b_{n-1} a_n$ and $\bar{b}_n = \bar{b}_{n-1} \bar{a}_n$ for $n = 1, 2, \ldots, 15$. and $b_0 = \bar{b}_0 = 1$. The received preamble, ignoring noise, is then

$$r_n = (1 + j)(b_n + \bar{b}_n).$$

The decision statistsic, $z$, is then calculated as

$$\begin{aligned}
z &= \sum_{n=1}^{15} a_n \mathrm{Re}(r^*_{n-1} r_n) \\
&= 2 \sum_{n=1}^{15} a_n (b_{n-1} + \bar{b}_{n-1})(b_n + \bar{b}_n) \\
&= 2 \sum_{n=1}^{15} a_n^2 + \bar{a}_n^2 + a_n b_{n-1} \bar{b}_n + a_n \bar{b}_{n-1} b_n \\
&= 2\left(15 + \sum_{n=1}^{15} \bar{b}_n b_n + \sum_{n=1}^{15} \bar{b}_{n-1} b_{n-1} + \sum_{n=1}^{15} a_n \bar{a}_n\right)
\end{aligned}$$

The first term in the above is the desired output, the last term is small due to orthogonallity between $a_n$ and $\bar{a}_n$, and the second and third terms are the undesired cross-terms. To within one summand, the cross-terms are crosscorrelations between the differentially encoded preambles which are generally not small.