TSG-RAN Working Group1 meeting #2 **TSGW1#2(99)101**
Yokohama 22-25, February 1999

Source: Hughes Network Systems and Nortel Networks

Title: A Low Complexity and Flexible Turbo Interleaver with Good Performance

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Abstract:
This document describes a merged turbo interleaver design technique which is flexible, simple to implement and has good performance.

1 Structure of Turbo Interleavers of Arbitrary Size:

We first describe turbo interleavers of arbitrary sizes. Design of mother interleavers with predefined depths are treated in Section 2.

Step 1: Given a desired turbo interleaver of size N, find the smallest m such that

$$2^m \geq N .$$

Step 2: Use the pre-designed interleaver of size 2^m, denoted as I_{2^m}, to form the new interleaver I_N by puncturing indices greater than or equal to N.

Example 1: Suppose we want to design an interleaver of size 5. Then m=3.
Suppose also that the pre-designed interleaver of size 2^m=8, i.e. I_8, is defined as

$$(0,1,2,3,4,5,6,7) \xleftarrow{I_8} (3,0,6,7,1,5,2,4)$$

Then the interleaver of size 5 is

$$(0,1,2,3,4) \xleftarrow{I_5} (3,0,1,2,4)$$

Therefore the interleaver design for any size N reduces to designing interleavers of size 2^m. In the following sections, we will describe how to design interleavers of size 2^m.

2 Structure of Turbo Interleavers of Size 2^m:

Turbo interleavers of size 2^m are formed using block interleavers with r rows and c columns where each row is permuted within itself using the following formula:

$$j \leftarrow k_i(j) \quad \text{where} \quad k_i(j) = (\alpha_i \times k_i(j-1) + \beta_i) \bmod c, \quad i = 0,1,2,\dots r-1 \quad j = 1,2,3,\dots, c-1$$

Here $k_i(j)$ denotes the original column position that gets mapped to the new column position j as a result of permutation and $k_i(0)=1$. α_i and β_i are integer constants specific for each row.

Before column permutation, the interleaver matrix is filled row by row After permutation the rows are interlaced based on bit reversal on row indices and finally the contents of the interleaver are read out column by column.

Example: Design an interleaver of size 32=4 x 8

Step 1: Fill in the interleaver row by row in the conventional way.

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 \\ 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 \end{bmatrix}$$

Step 2: Permute each row i (i=0,1,2,3) according to the formula,

$$j \leftarrow k_i(j) \quad \text{where} \quad k_i(j) = (\alpha_i \times k_i(j-1) + \beta_i) \bmod c, \quad i = 0,1,2,3 \quad j = 1,2,\dots,6$$

For the sake of this example, assume that the following constants α_i and β_i have been found to yield optimal turbo interleavers of size N, where $16 < N \leq 32$.

I	α_i	β_i
0	1	7
1	5	3
2	5	1
3	1	5

The shuffling of each row can then be found as,

row 0: $(0,1,2,3,4,5,6,7) \leftarrow (1,\ 0,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2)$
row 1: $(0,1,2,3,4,5,6,7) \leftarrow (1,\ 0,\ 3,\ 2,\ 5,\ 4,\ 7,\ 6)$
row 2: $(0,1,2,3,4,5,6,7) \leftarrow (1,\ 6,\ 7,\ 4,\ 5,\ 2,\ 3,\ 0)$
row 3: $(0,1,2,3,4,5,6,7) \leftarrow (1,\ 6,\ 3,\ 0,\ 5,\ 2,\ 7,\ 4)$

Applying the above shuffling to the rows of the interleaver matrix yields:

$$\begin{bmatrix} 1 & 0 & 7 & 6 & 5 & 4 & 3 & 2 \\ 9 & 8 & 11 & 10 & 13 & 12 & 15 & 14 \\ 17 & 22 & 23 & 20 & 21 & 18 & 19 & 16 \\ 25 & 30 & 27 & 24 & 29 & 26 & 31 & 28 \end{bmatrix}$$

Step 3: Re-order the rows according to the bit reversal on row index (00,01,10,11),

$$\begin{bmatrix} 1 & 0 & 7 & 6 & 5 & 4 & 3 & 2 \\ 17 & 22 & 23 & 20 & 21 & 18 & 19 & 16 \\ 9 & 8 & 11 & 10 & 13 & 12 & 15 & 14 \\ 25 & 30 & 27 & 24 & 29 & 26 & 31 & 28 \end{bmatrix}$$

Step 4: Read out the contents of the interleaver column by column,

1 17 9 25 0 22 8 30 7 23 11 27 6 20 10 24 5 21 13 29 4 18 12 26 3 19 15 31 2 16 14 28

3 Search for Optimal Turbo Interleavers

In this section, we report the results of our search for the "best" prunable mother interleavers of size 256, 512, 1024, 2048, 4096 and 8192 so that they yield best interleavers of any size N where $128 < N \leq 8192$. As described in Section 2, each of these interleavers of size 2^m is formed using block interleavers where each row is shuffled within itself according to constants α_i and β_i. These constants are given in Table 1. For simplicity, only four different α is used for each mother interleaver.

Table 1: Parameters for the Prunable Interleavers

Row index i	256=8x32 α_i	β_i	512=16X32 α_i	β_i	1024=16X64 α_i	β_i	2048=32X64 α_i	β_i	4096=32X128 α_i	β_i	8192=64X128 α_i	β_i
0	25	11	9	11	33	21	33	31	105	39	81	63
1	17	27	25	19	49	13	17	27	37	19	49	93
2	9	3	5	27	1	55	13	25	117	79	77	101
3	1	3	17	31	17	19	29	25	17	79	33	63
4		27		5		27		51		23		39
5		11		21		29		3		83		29
6		11		15		11		3		113		19
7		31		29		35		27		59		99
8				19		1		45		45		27
9				5		5		9		3		61
10				23		13		7		65		61
11				5		43		37		75		13
12				21		63		27		123		57
13				13		61		61		43		23
14				9		17		63		15		5
15				15		3		63		3		65
16								47		67		15
17								1		17		117
18								17		91		9
19								49		17		11
20								1		99		29
21								41		79		111
22								1		95		59
23								7		125		81
24								63		85		11
25								51		101		55
26								19		103		49
27								29		87		111
28								49		13		81
29								35		49		3
30								43		111		95
31								35		123		41
32												101
33												49
34												7
35												93
36												83
37												113

38												57
39												29
40												25
41												125
42												117
43												37
44												51
45												1
46												93
47												7
48												3
49												27
50												105
51												17
52												125
53												73
54												41
55												67
56												95
57												9
58												45
59												69
60												35
61												41
62												3
63												75

4 Simulation Results

Simulation results with interleaver sizes of 320 and 640 are given in Figures 1 and 2. The performance is comparable to that of MIL interleaver.

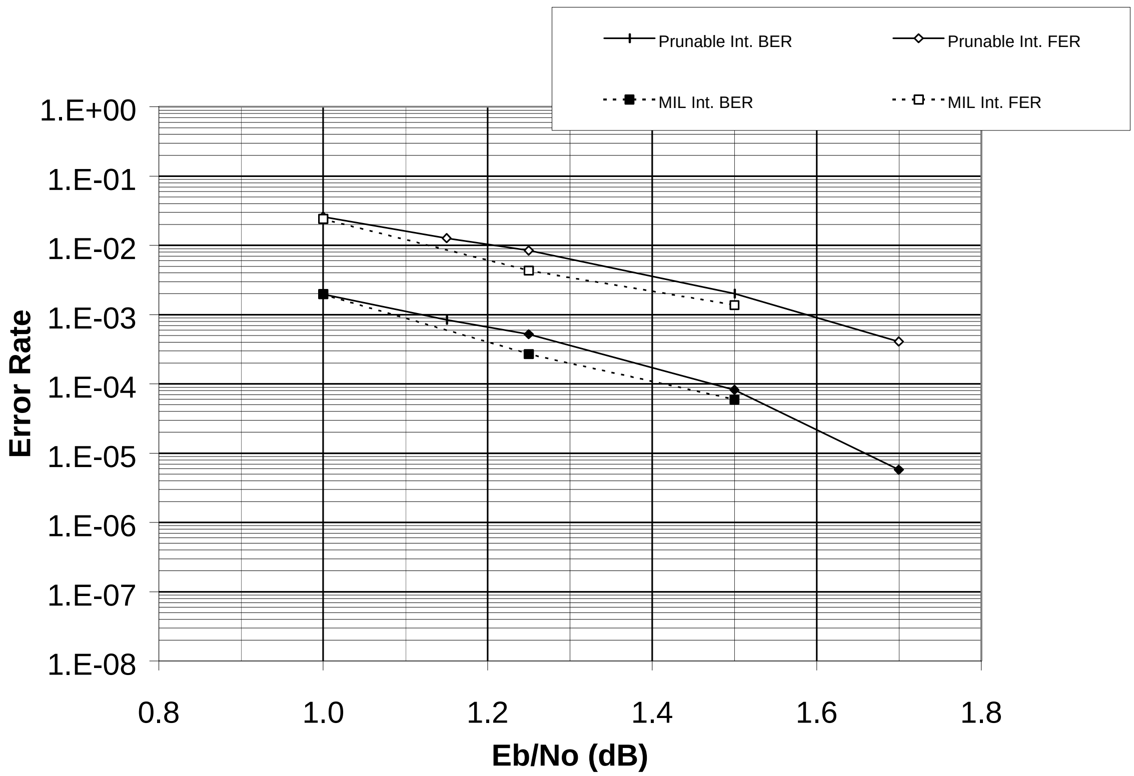

Figure 1: Performance of Turbo Interleavers, size=320, AWGN

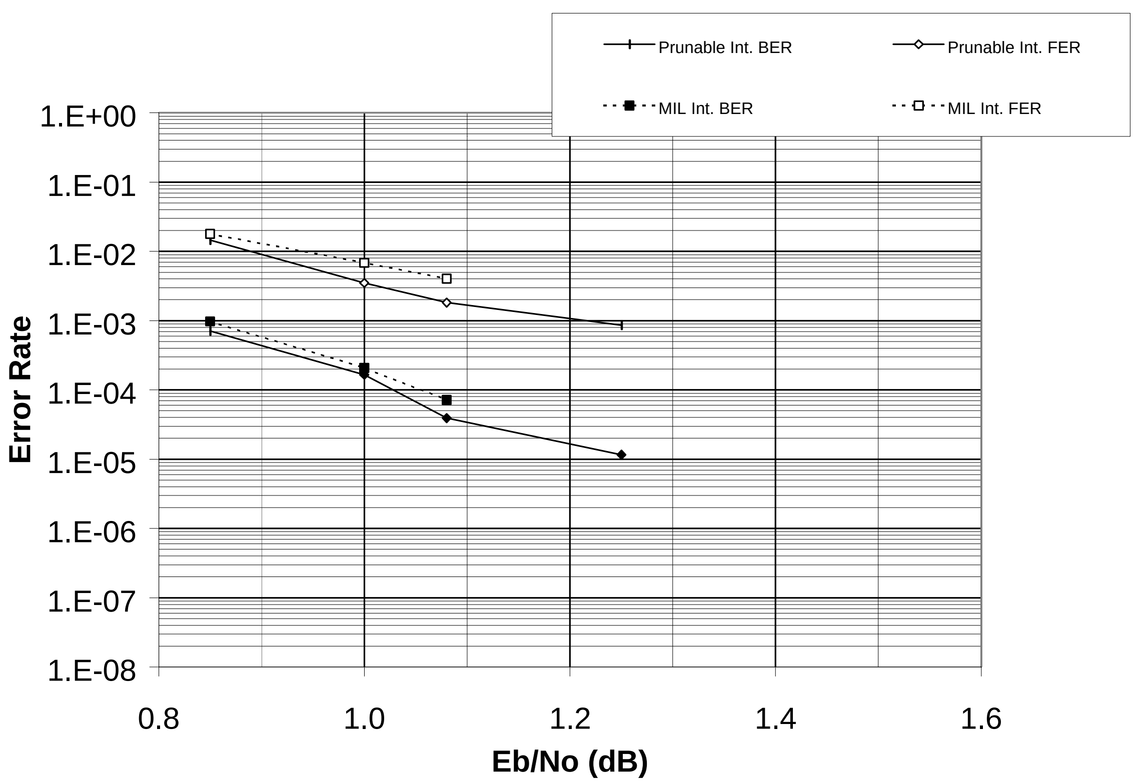

Figure 2: Performance of Turbo Interleavers, size=640, AWGN

5 Conclusion

A simple turbo interleaver with good performance is described. There are only a few constants that need to be stored for the entire range of interleaver sizes and the interleaved indices can be very easily generated on the fly. Moreover the number of consecutive puncturing is no more than one.