Title:	Modeling Intercell Interference
File:	SCM-084-LUC-intercell_interf
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Date:	December 19, 2002

1. INTRODUCTION

In previous contributions on the topic of modeling intercell interference [1][2], a threshold was fixed and bases whose received power exceeded the threshold had their spatial interference explicitly modeled. Bases whose received power was below this threshold had their interference modeled as spatially white noise. Using this threshold method, the number of bases to be explicitly modeled varies depending on the realizations of the shadow fading. In this contribution, we propose a different technique where we fix the number of bases that we explicitly model. As a result, the simulation complexity is fixed for each drop. We provide recommend text for inclusion in the SCM text based on this technique for modeling intercell interference.

2. METHODOLOGY

Because the methodology was covered previously in [1] and [2], we review it quickly here and emphasize the differences with the newly proposed technique.

We consider the received signal by a mobile receiver with R antennas in the center cell of a system with 2 rings of hexagonal cells, each with 3 sectors for a total of 57 sectors. Each of the bases has T transmit antennas. As in [3], the received powers from all 57 sectors are determined based on path loss and shadow fading, and the sector with the strongest power is chosen to be the serving sector. Let the received signal for a given chip interval be given by

$$\mathbf{r} = \sum_{i=0}^{56} \frac{A_i}{\sqrt{T}} \mathbf{H}_i \mathbf{b}_i \tag{1}$$

where A_i (i = 0, 1, …, 56) is the amplitude (per transmit antenna) of the signal from the ith sector, $\mathbf{H}_i$ is the R-by-T channel matrix corresponding to this sector, $\mathbf{b}_i$ is the T-dimensional vector corresponding to the chip elements of the T transmitted signals from this sector. For simplicity, we ignore the presence of additive white Gaussian noise. The elements of the matrices $\mathbf{H}_i$ are i.i.d., complex Gaussian random variables with unit power. In general, the elements of these matrices are derived from the spatial channel model. The elements of $\mathbf{b}_i$ have unit power, and the indices are ordered such that $A_0 > A_1 > A_2 > \ldots > A_{56}$. Given the received signal, the MMSE receiver is the R-by-T matrix [2]:

$$\mathbf{W} = \frac{\sqrt{T}}{A_0} \mathbf{H}_0 \left(\mathbf{H}_0^H \mathbf{H}_0 + \sum_{i=1}^{56} \frac{A_i^2}{A_0^2} \mathbf{H}_i^H \mathbf{H}_i \right)^{-1} \tag{2}$$

Suppose we wish to spatially model the interference from only a subset of the bases. Specifically, we denote the set $F(M)$ of those bases corresponding to the M largest amplitudes of the interfering bases: $A_1, A_2, \ldots, A_M$. In order to approximate the spatial interference from weaker bases, we compute the covariance of the signal from the ith base as

$$E\left[\frac{A_i^2}{T} \mathbf{H}_i \mathbf{b}_i \mathbf{b}_i^H \mathbf{H}_i^H \right] = \frac{A_i^2}{T} E\left[\mathbf{H}_i \mathbf{H}_i^H \right] = \frac{A_i^2}{T} \begin{bmatrix} T & & \\ & \ddots & \\ & & T \end{bmatrix} = A_i^2 \mathbf{I}_R \tag{3}$$

where $\mathbf{I}_R$ is the R-by-R identity matrix. Assuming that the interference from bases outside of set $F(M)$ can be modeled as spatially white Gaussian noise, the received signal in (1) can be rewritten as

$$\mathbf{r} = \frac{A_0}{\sqrt{T}}\mathbf{H}_0\mathbf{b}_0 + \sum_{i\in F(M)}\frac{A_i}{\sqrt{T}}\mathbf{H}_i\mathbf{b}_i + \sum_{i\notin F(M)} A_i\mathbf{n}_i \tag{4}$$

where $\mathbf{n}_i$ is a complex Gaussian random vector with zero mean and variance $\mathbf{I}_R$. Therefore the MMSE receiver which models the spatial interference from only the set $F(M)$ explicitly is

$$\mathbf{W}(M) = \frac{\sqrt{T}}{A_0}\mathbf{H}_0\left(\mathbf{H}_0^H\mathbf{H}_0 + \sum_{i\in F(M)}\frac{A_i^2}{A_0^2}\mathbf{H}_i^H\mathbf{H}_i + \sum_{i\notin F(M)}\frac{TA_i^2}{A_0^2}\mathbf{I}_T\right)^{-1}. \tag{5}$$

Note that for w = 56, $\mathbf{W}(M)$ in (2) is equivalent to $\mathbf{W}$ in (1). We write the matrices $\mathbf{H}_0$ and $\mathbf{W}(M)$ in terms of their column vectors $\mathbf{H}_0 = \left[\mathbf{h}_{0,1}\cdots\mathbf{h}_{0,T}\right]$ and $\mathbf{W}(M) = \left[\mathbf{w}_1(M)\cdots\mathbf{w}_T(M)\right]$, and we write the data vector $\mathbf{b}_0$ in terms of its components $\mathbf{b}_0 = [b_{0,1}\ldots b_{0,T}]^T$. The output of the MMSE receiver for the tth antenna ($t = 1 \ldots T$) is the inner product between the column vector $\mathbf{w}_t(M)$ and the received signal $\mathbf{r}$ given by (1):

$$\mathbf{w}_t^H(M)\mathbf{r} = \frac{A_0}{\sqrt{T}}\mathbf{w}_t^H(M)\mathbf{h}_{0,t}b_{0,t} + \frac{A_0}{\sqrt{T}}\sum_{j=1,j\neq t}^{T}\mathbf{w}_t^H(M)\mathbf{h}_{0,j}b_{0,j} + \sum_{i=1}^{56}\frac{A_i}{\sqrt{T}}\mathbf{w}_t^H(M)\mathbf{H}_i\mathbf{b}_i \tag{6}$$

The SINR for the tth antenna ($t = 1 \ldots T$) can be computed from (6):

$$SINR_{t,full}(M) = \frac{A_0^2\left|\mathbf{w}_t^H(M)\mathbf{h}_t\right|^2}{A_0^2\sum_{\substack{j=1\\ j\neq t}}^{T}\left|\mathbf{w}_t^H(M)\mathbf{h}_j\right|^2 + \sum_{i=1}^{56}A_i^2\left\|\mathbf{w}_t^H(M)\mathbf{H}_i\right\|^2}, \tag{7}$$

where the *full* subscript denotes the fact that the spatial interference was fully modeled in computing the SINR. The derivation of the MMSE receiver and SINR up to this point was already given in [1]. We now extend the evaluation of SINR to the case where the interference term in the denominator accounts for the spatial interference characteristics for only set $F(M)$. In other words, we write the out put of the MMSE receiver assuming that the received signal is given by (4):

$$\mathbf{w}_t^H(M)\mathbf{r} = \frac{A_0}{\sqrt{T}}\mathbf{w}_t^H(M)\mathbf{h}_{0,t}b_{0,t} + \frac{A_0}{\sqrt{T}}\sum_{j=1,j\neq t}^{T}\mathbf{w}_t^H(M)\mathbf{h}_{0,j}b_{0,j} + \sum_{i=1}^{56}\frac{A_i}{\sqrt{T}}\mathbf{w}_t^H(M)\mathbf{H}_i\mathbf{b}_i \tag{8}$$

The SINR for the tth antenna ($t = 1 \ldots T$) can be computed from (8):

$$SINR_{t approx}(M) = \frac{A_0^2\left|\mathbf{w}_t^H(M)\mathbf{h}_t\right|^2}{A_0^2\sum_{\substack{j=1\\ j\neq t}}^{T}\left|\mathbf{w}_t^H(M)\mathbf{h}_j\right|^2 + \sum_{i\in F}A_i^2\left\|\mathbf{w}_t^H(M)\mathbf{H}_i\right\|^2 + \sum_{i\notin F}A_i^2\left\|\mathbf{w}_t^H(M)\right\|^2}, \tag{9}$$

where *approx* denotes that the spatial interference is modeled approximately. In the next section, we refer to the SINR expressions in equations (7) and (9) as “full SINR” and “approximate SINR,” respectively.

3. NUMERICAL RESULTS

We compute the SINR in equations (7) or (9) for N channel realizations, and we denote the nth realization (n = 1 … N) as $SINR_{t,full}(M,n)$ and $SINR_{tapprox}(M,n)$. We compute N = 10000 realizations of $SINR_{t,full}(M,n)$ and $SINR_{tapprox}(M,n)$ (M = 4, 8, 12) by randomly placing a user uniformly in the center cell of a 19-cell, two-ring hexagonal cell configuration. We let

$$e_{tfull}(M,n) := \left| \frac{SINR_{tfull}(M,n) - SINR_{t,full}(56,n)}{SINR_{tfull}(56,n)} \right| \tag{10}$$

be the normalized error between $SINR_{t,full}(M,n)$ and $SINR_{t,full}(56,n)$ for the nth realization. Similarly, we let

$$e_{t,approx}(M,n) := \left| \frac{SINR_{t,approx}(M,n) - SINR_{tfull}(56,n)}{SINR_{tfull}(56,n)} \right| \tag{11}$$

be the normalized error between $SINR_{tapprox}(M,n)$ and $SINR_{t,full}(56,n)$ for the nth realization. Note that the error is always measured with respect to $SINR_{t,full}(56,n)$, the calculation of SINR using the full modeling of the spatial interference for both the MMSE receiver and the interference in the SINR.

We also compute the ratio of the total “unmodeled” interference power to the total interference power:

$$\frac{\sum_{i=F+1}^{56} A_i^2}{\sum_{i=1}^{56} A_i^2}.$$

In Figures 3 and 4, we denote this quantity to be the power ratio.

We collect the errors for all t = 1 … T and n = 1 … N and plot its cumulative distribution function (CDF) in Figure 1 for spatially uncorrelated channels, and in Figure 2 for spatially correlated channels (with the channel parameters given in Table 1). In Figure 1, starting from the left, the CDF of the error for M = 8 is less than 1% about 85% of the time using the full SINR model in equation (7) and the error expression in (10). Simplifying the SINR calculation using the approximate SINR in equation (9) and the error expression in (11), the overall error increases and is less than 1% about 55% of the time. Therefore by ignoring the spatial structure of 56 – 8 = 48 bases, the error in the SINR output is minimal for a significant fraction of time. As seen in Figure 2, the SINR errors for the correlated channel for M = 8 are basically the same as for the uncorrelated channel.

Figure 3 shows that for M = 8, the ratio of the unmodeled to total interference power is less than 2% for 90% of the time. Hence the received power of those bases which are approximated as white noise is always less than a factor 0.0064 of the received power of the serving base. The same result is seen for the correlated channel case in Figure 4.

We feel that setting M = 8 provides the proper balance between complexity and SINR accuracy. Furthermore by setting M = 8, the unmodeled interference power is less than 2% of the total interference power for a significant majority (90%) of the cases.

BS antenna separation	4 wavelengths
MS antenna separation	0.5 wavelengths
BS PAS, angle spread	Laplacian, 35 degrees
MS PAS, angle spread	Laplacian, 5 degrees

Table 1. Parameters of spatially correlated channels

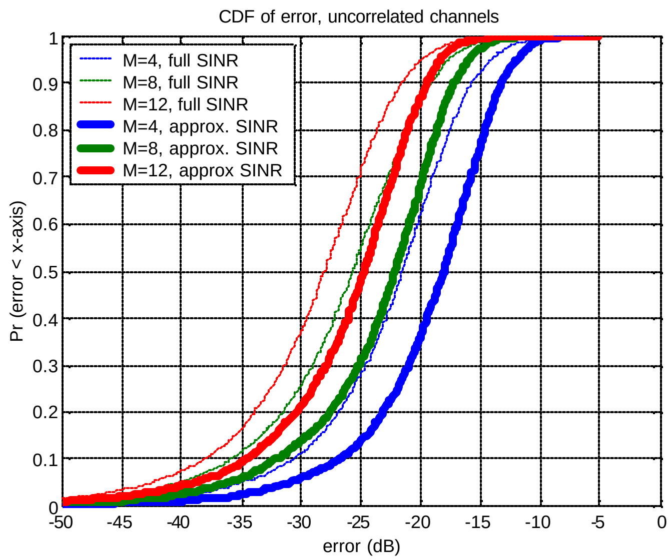

Figure 1. CDF of error, uncorrelated channels

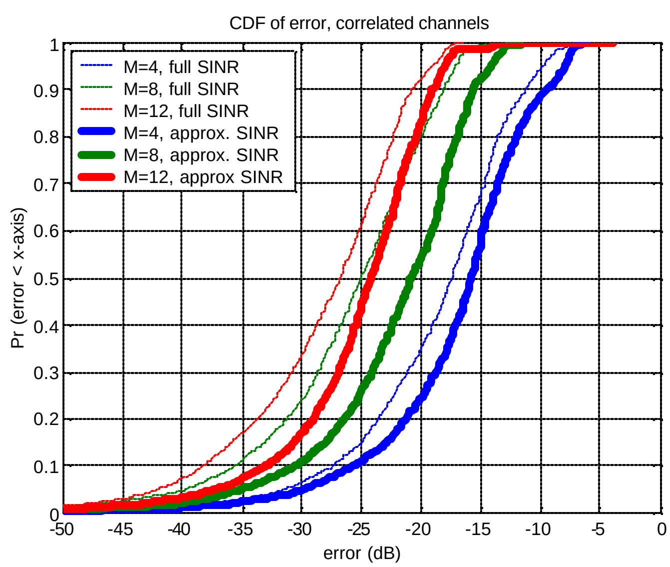

Figure 2. CDF of error, correlated channels

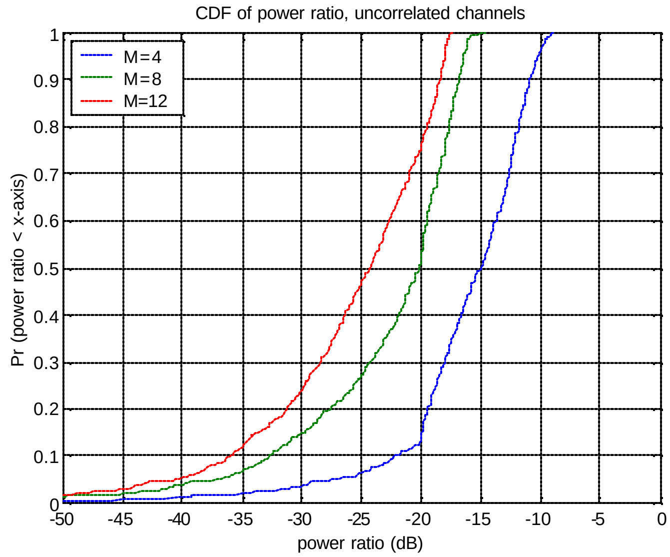

Figure 3. CDF of threshold, uncorrelated channels

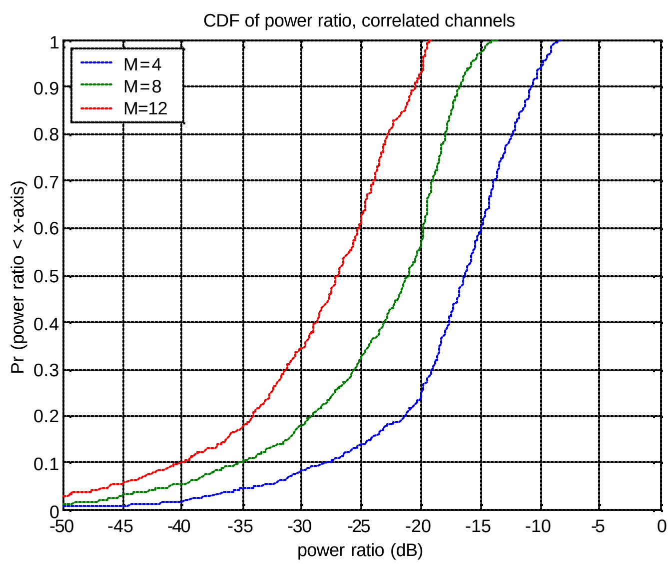

Figure 4. CDF of number of threshold, correlated channels

4. CONCLUSIONS AND TEXT PROPOSAL

We propose a threshold-based technique for determining how to account for intercell interference. We show that by modeling only a small fraction of the strongest interferers spatially, the system simulation complexity can be reduced significantly without significantly impacting the resulting SINR measurement at the output of an MMSE detector. Based on these observations, we propose that the system simulation methodology follow a similar technique to account for intercell interference, and we propose the following text to be inserted into the SCM text:

To simplify the system simulations, the spatial interference should be modeled explicitly for only a fraction of the transmitting bases. Let A_0 *denote the received amplitude of the serving base based on shadow fading and pathloss, and let* $A_1, A_2, \ldots, A_B$ *denote the received amplitudes of the remaining B bases. For example, in a system with 2 concentric rings of hexagonal cells, there are a total 19 cells; and if each cell is split into 120 degree sectors, there are a total of B = 3*19 – 1 interfering bases. We further assume that the indices are assigned such that* $A_0 > A_1 > A_2 > \ldots > A_B$. *The spatial interference shall be modeled explicitly for only the M = 8 strongest interferers, and the spatial interference of the remaining B-M bases shall be modeled as spatially white noise. In other words, only those bases which correspond to amplitudes* $A_1, A_2, \ldots, A_8$ *shall be modeled explicitly.*

Additional text can be added to clarify the modeling once the system simulation notation is defined.

5. REFERENCES

[1] SCM-071, Lucent, “Modeling intercell interference”

[2] SCM-075, Lucent, “Modeling intercell interfernece.”