TSG-RAN Working Group 1 meeting #5
Cheju, South Korea
June 1-4, 1999

3GPP/TSGR1#5(99)670

TITLE:

Evaluation of Proposed RACH Signatures

SOURCE:

Motorola

1.0 Introduction

Two new proposals for the RACH preamble codes have recently been presented [1][2]. Presently, the RACH preamble codes consist of length 256 orthogonal Gold spreading codes concatenated with length 16 orthogonal Gold sequences. In [1] a similar concatenated approach based on complementary Golay sequences which allow efficient matched filtering implementations and have improved autocorrelation properties was presented. Nokia has suggested using a long code of length 4096 and a set of orthogonal sequences of length 16 [2]. This contribution addresses several issues with the present preambles based on Gold codes and the proposed preambles based on complementary Gold sequences. The Nokia proposal was not included due to lack of technical detail although the issue raised are discussed in terms of random codes which may be a reasonable approximation to the Nokia approach.

2.0 Cross-Correlation Properties with Large Time Offsets

The original preamble sequences based on Gold sequences and the sequences presented in [1] consist of 16 complex symbols spread with length 256 orthogonal spreading codes. This construction allows an efficient implementation of the bank of correlators required for detection when the correlations are performed at offsets within a 255 chip window. A round trip delay of 255 chips corresponds to a cell radius of approximately 9 km. Larger cell sizes would require additional processing such as additional correlator banks.

The autocorrelation properties of both Golay and Gold preamble codes were described in [1] in terms of the maximum absolute aperiodic autocorrelation sidelobes for lags in both the 256 and 4096 chip ranges. Crosscorrelation properties were later distributed on the reflector for offsets through 255 chips. Systems deployed in rural areas with large antenna heights and systems which cover large areas of water will however require cells with radii larger than 9 km. It is therefore important to evaluate the crosscorrelation properties of the proposed signatures at lags larger than 255 chips.

Golay Codes

Figure 1 shows a plot of the crosscorrelation between signatures 0 and 5 in Table 1 of [1] in the range -511 to 511 chips. Two sidelobes of magnitude 2048 are seen at offsets of +/- 256 chips.

Large sidelobes like these can occur with the proposed Golay codes at offsets of multiples of 256 chips because at these offsets, the length 256 spreading codes align and are either orthogonal or identical. This is illustrated in Table 1 for the example of Fig. 1.

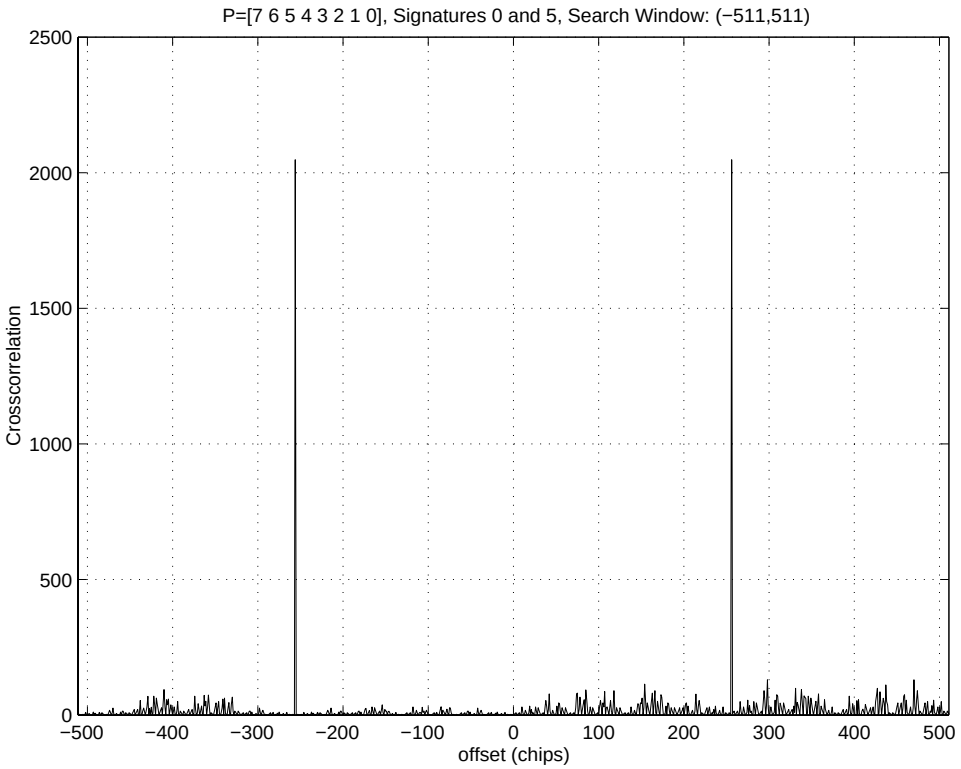

Figure 1: Crosscorrelation between signatures 0 and 5 for the Golay codes. One particular length 256 spreading code was used.

A	A	B	B	A	-A	-B	B	A	-A	B	-B	A	A	-B	-B	
	A	A	B	B	-A	A	B	-B	-A	A	-B	B	A	A	-B	-B
0	256	0	256	0	256	0	256	0	256	0	256	0	256	0	256	0

Table 1: Calculation of crosscorrelations between signatures 0 and 5

The large sidelobes in the above example actually occur for all signatures. Table 2 shows the crosscorrelation magnitudes evaluated at lags equal multiples of 256 chips between signature k and signatures $l \in \{0, 1, \cdots, 15;\ l \neq k\}$ for search windows through 2304 chips. Also shown is the maximum cell radius for each window size. These results are independent of the choice of signature k.

Figures 2 and 3 show histograms of the maximum absolute crosscorrelations for offsets through 255 and 1023 chips respectively. Each point represents the maximum absolute correlation over

Cell radius (km)	Maximum round trip delay (μs)	Search Window (chips)	Crosscorrelation Magnitude	Number of Occurrences
9.4	62.5	[–256, 256]	2048	4
18.8	125	[-512,512]	2048	4
28.1	187.5	[–768, 768]	512	12
			1536	4
			2048	4

Table 2: Crosscorrelation magnitudes at 256 chip offsets for the Golay sequences.

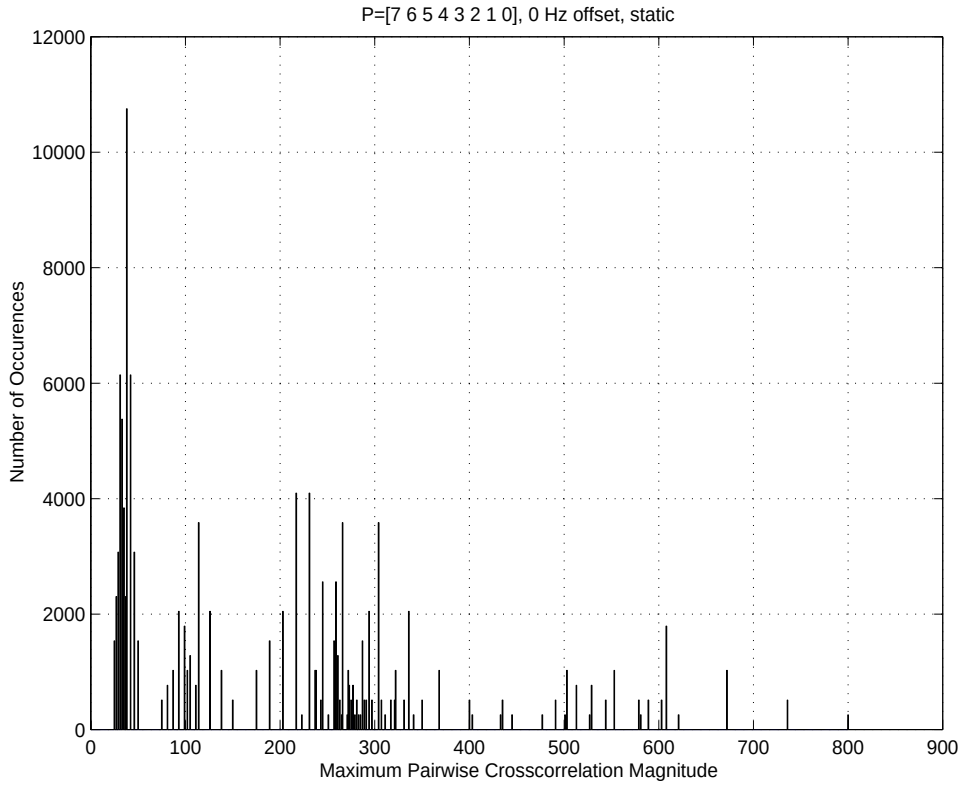

Figure 2: Crosscorrelation magnitudes with a 255 chip window for the Golay codes

the set of offsets for a particular pair of signatures. There are therefore

$$256 \cdot \sum_{n=2}^{32} n(n-1) = (256 \cdot 33 \cdot 32)/2 \quad (1)$$

points in all.

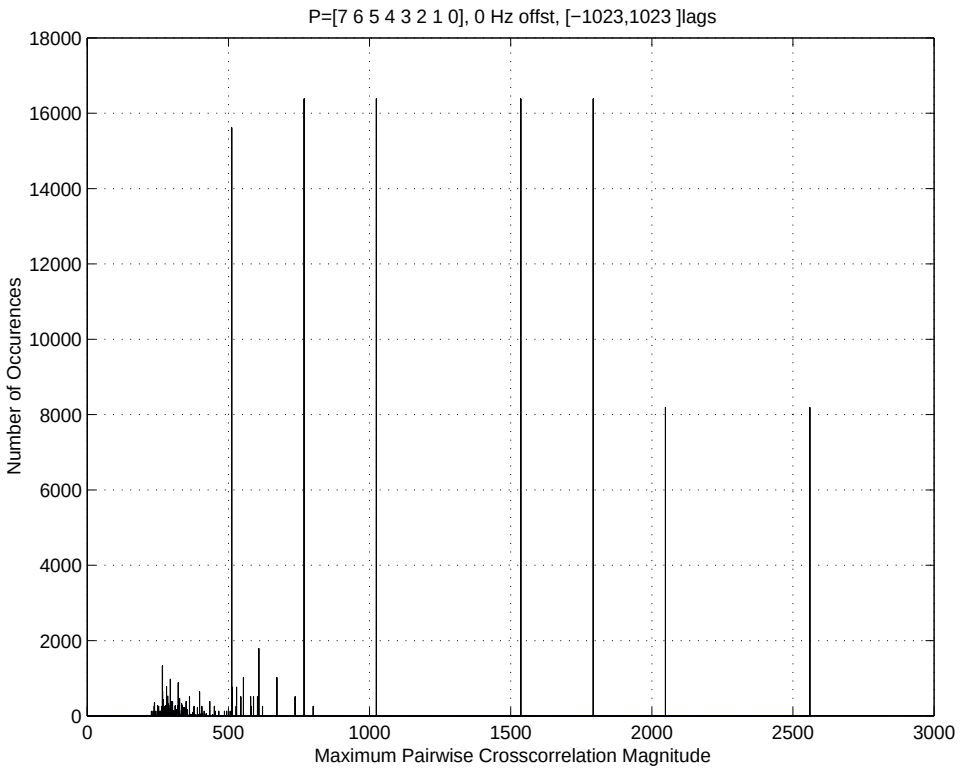

Figure 3: Crosscorrelation magnitudes within a 1023 chip window for the Golay codes.

Gold Codes

A table of crosscorrelation magnitudes corresponding to Table 1 was calculated for the case of Gold codes. As for the Golay codes, the set of crosscorrelation magnitudes was independent of the signature k. However, the number of occurrences of each magnitude did vary with signature and therefore Table 2 contains the average number (over 16 signatures) of occurrences of each magnitude. Overall the crosscorrelation performance of the codes is similar. Both have maximum correlations of 2048 within +/- 512 search windows although the Gold codes have, on average, lower multiplicities of these events. The Gold codes however have a larger number of correlations in the 1024 through 1792 range.

More importantly both preamble codes have significant extraneous peaks which can cause false signature detections resulting in a number of problems. These include missed detections, wasted base resources used in message decoding, premature acknowledgements of signatures with corrupted correlator outputs, and wasted AICH power from erroneous acknowledgements.

Random codes which take on +/- 1 with equal probability will have a distribution of crosscorrelations with standard deviation equal to the square root of the number of chips integrated. The +/- 2σ points therefore are at most +/ 128.

3.0 Crosscorrelation Degradation Due to Channel Phase Rotation

A second problem with the Golay and Gold codes is the sensitivity of their correlation structure to channel phase rotation. Channel phase rotation occurs when there is a frequency offset between

Cell radius (km)	Maximum round trip delay (μs)	Search Window (chips)	Crosscorrelation Magnitude	Average Number of Occurrences
9.4	62.5	[–256, 256]	256	5.6
			768	12.9
			1280	9.6
			1792	1.9
18.8	125	[–512, 512]	256	5.6
			512	11.3
			768	12.9
			1024	3.8
			1280	9.6
			1536	3.8
			1792	1.9
			2048	2.8
28.2	187.5	[–768, 768]	256	15
			512	11.3
			768	24.4
			1024	3.8
			1280	16.9
			1536	3.8
			1792	3.8
			2048	2.8

Table 3: Crosscorrelation magnitude at 256 chip offsets for the Gold codes.

the transmitted and received signatures. This frequency offset may have two causes. First, there may be a difference between the received BS carrier frequency and the MS transmitted signal. This for example could be due to the frequency error in the mobile's AFC (automatic frequency control). Second, MS movement will cause a Doppler shift in proportion to the vehicle's speed. Table 4 shows these shifts for several vehicle speeds. Note that, depending on mobile implementation, the total frequency offset due to Doppler could be twice that listed in Table 4 if frequency synchronization is achieved by locking the mobile's transmitt frequency to the frequency of the

signal received from the base.

Vehicle Speed	Frequency Offset (Hz)
30 km/h	56
120 km/h	222
200 km/h	370
500 km/h	925

Table 4: Frequency offset due to Doppler shift. Carrier frequency is 2 GHz.

The presence of frequency offset degrades the correlation properties of the set of signatures. The change in cross-correlation due to frequency offset can be explained as follows. If the transmitted signature is $\mathbf{s}_k = [s_{1k}, s_{2k}, \cdots, s_{Nk}]^T$ and the channel is rotating θ radians per symbol, then the signature received at the base is a modified version of the transmitted signature, $\hat{\mathbf{s}}_k$

$$\hat{s}_{nk} = e^{j(n-1)\theta} s_{nk} . \tag{2}$$

For simplicity the initial phase is assumed to be zero and the phase rotation during a symbol is assumed negligible. The cross-correlation between the received signature and signature l is then

$$\langle \hat{\mathbf{s}}_k, \mathbf{s}_l \rangle = e^{j\theta} \sum_{n=1}^{N} s_{nk} e^{jn\theta} s_{nl}^* \quad . \tag{3}$$

Multiplication of the transmitted signature by an exponential with linearly changing phase modifies the signature so that it becomes “similar” to, or correlated with, other signatures. As a simple example consider two length four signatures. The first being 1 1 -1 -1 and the second being 1 1 1 1. If the first signature is transmitted through a channel whose phase rotates from 0 degrees at the beginning of the first symbol through 360 degrees at the end of the last, then the polarity of the last two symbols will be flipped thereby making it highly correlated with the signature of all 1’s.

Golay Codes

Something similar to the above example occurs with the Golay sequences in that the last eight symbols of signatures zero and one in Table 1 of [1] have opposite polarity. If the channel rotates one period over the duration of the signature, i.e, 1000 Hz, the first signature will be highly correlated with the second. This is seen in Fig. 4 where the magnitude square of the correlation between signatures as defined by (3) are plotted for offsets between 100 and 1200 Hz for an assumed transmission of signature 1. With an offset of 400 Hz, the correlation with the second signature is about 3 dB down from that of the first.

Similar degradations occur in the spectrum of crosscorrelations between time-shifted signatures.

Figure 6 is a histogram of the maximum absolute crosscorrelations for offsets through 255 chips with a 400 Hz frequency offset. Comparing Figs. 2 and 6 we see an increase in the maximum correlation from to 750 to 2300. Correlations through 1023 chips with a 400 Hz offset are shown in Fig. 7. Comparison with Fig. 3 indicates an overall increase in crosscorrelation when offsets through 1023 chips are considered.

Gold Codes

Magnitude square correlations vs. offset frequency is plotted in Fig. 5 for Gold preambles. Comparing Figs. 4 and 5 indicates slightly less sensitivity to frequency offset for the Gold codes. At 600 Hz offset, signature 14's correlation is almost as large as that of the transmitted signal.

The crosscorrelation between random codes are not as sensitive to offset frequency as the Golay and Gold codes. If $\mathbf{s}_k$ and $\mathbf{s}_l$ in (3) are taken to be random signatures, then the resulting complex crosscorrelation defined by that equation has a variance which is independent of the offset frequency.

4.0 Crosscorrelation with Multiple Doppler Channels

Large frequency offsets have been reported to significantly degrade the detection probability of conventional correlation detection of signatures [3]. One method of improving this probability is suggested in [5]. In this approach correlations are performed not only over hypothesized time offsets and signatures but also over hypothesized offset frequencies. While this method can compensate for a loss of detection probability when large offsets are present, it worsens the crosscorrelation problems discussed in the previous section. The method consists of performing correlations of the form

$$\langle \mathbf{s}_l, \mathbf{r}, mf_\delta \rangle = \sum_{n=0}^{N-1} s_{nl}^* e^{-j2\pi f_\delta T_s mn} r_n \tag{4}$$

for a set of frequencies mf_δ, $m = -M, -M+1, \cdots, 0, M-1, M$. The vector $\mathbf{r}$ in the above equation is the received vector of symbols. If the received signal consists of a single preamble, s_k, with offset frequency f_o, then these correlations can be expressed as

$$\langle \mathbf{s}_l, \mathbf{s}_k, mf_\delta \rangle = \sum_{n=0}^{N-1} s_{nl}^* e^{-j2\pi n T_s (mf_\delta - f_o)} s_{nk} \quad . \tag{5}$$

By comparing of (5) with (3) it can be seen that correlating over multiple Doppler channels makes the crosscorrelation problem discussed in Section 3 worse. Signature k is now modified by the exponential $e^{-j2\pi n T_s (mf_\delta - f_o)}$ instead of $e^{-j2\pi n T_s f_o}$. For $m < 0$ the effective channel rotation has increased and therefore crosscorrelation between signatures increases.

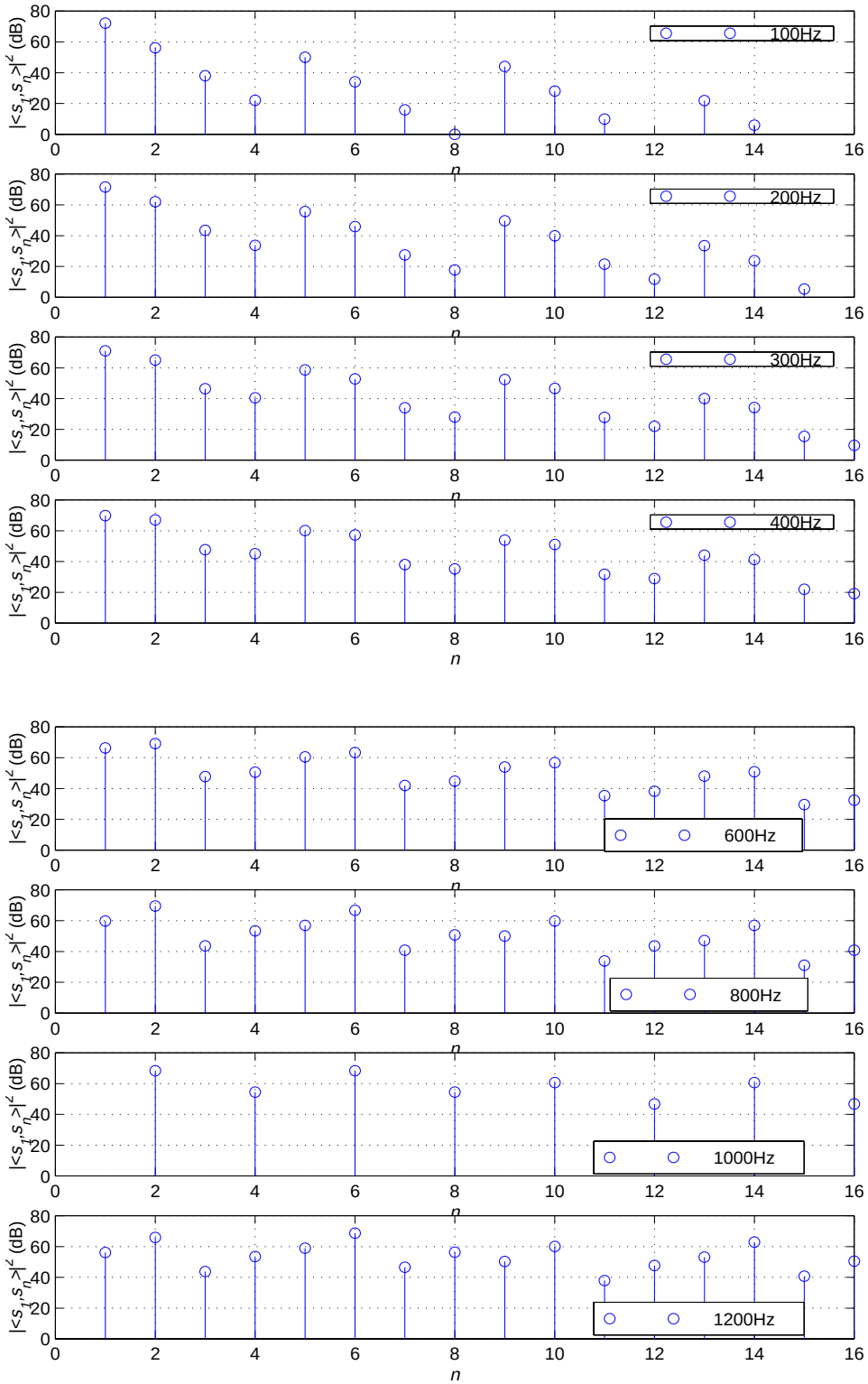

Figure 4: Crosscorrelation magnitudes vs. frequency offset for signatures based on Golay codes. Signature 1 transmitted.

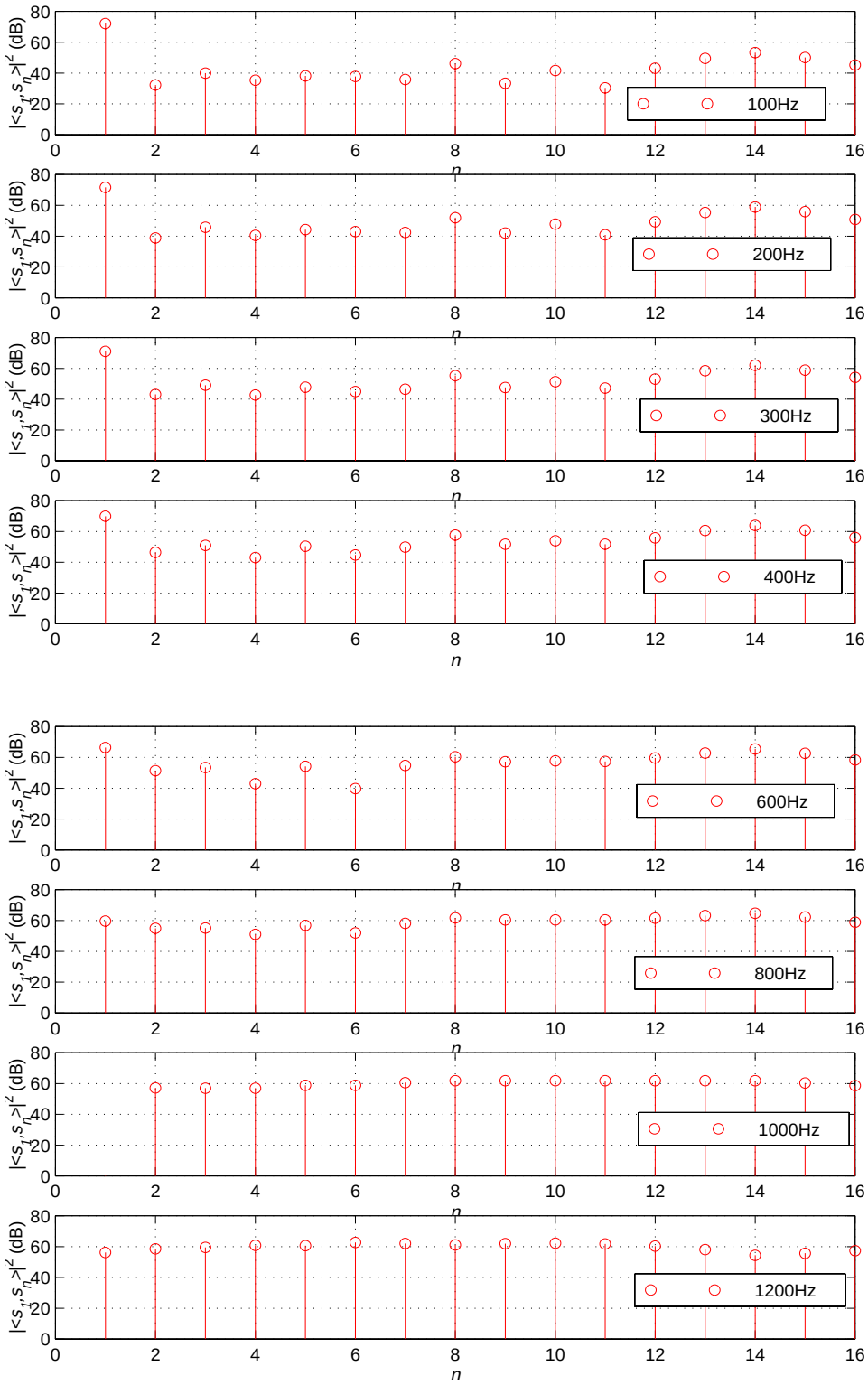

Figure 5: Crosscorrelation magnitudes vs. frequency offset for signatures based on Gold codes. Signature 1 transmitted.

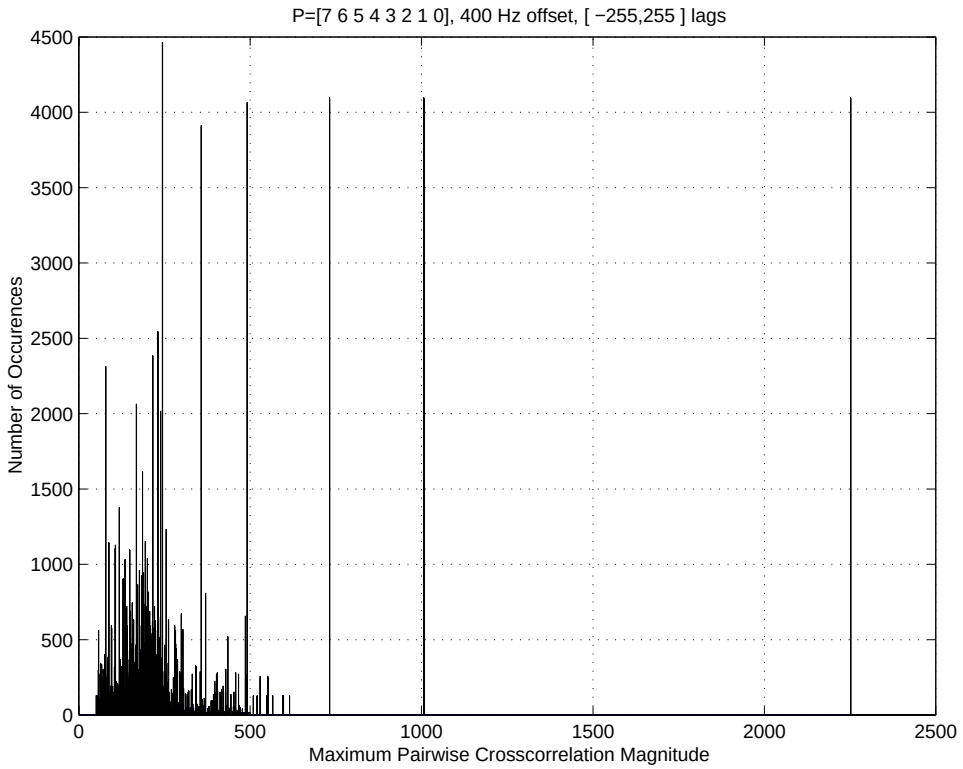

Figure 6: Crosscorrelation for 255 chip search window and 400 Hz frequency offset: Golay Codes.

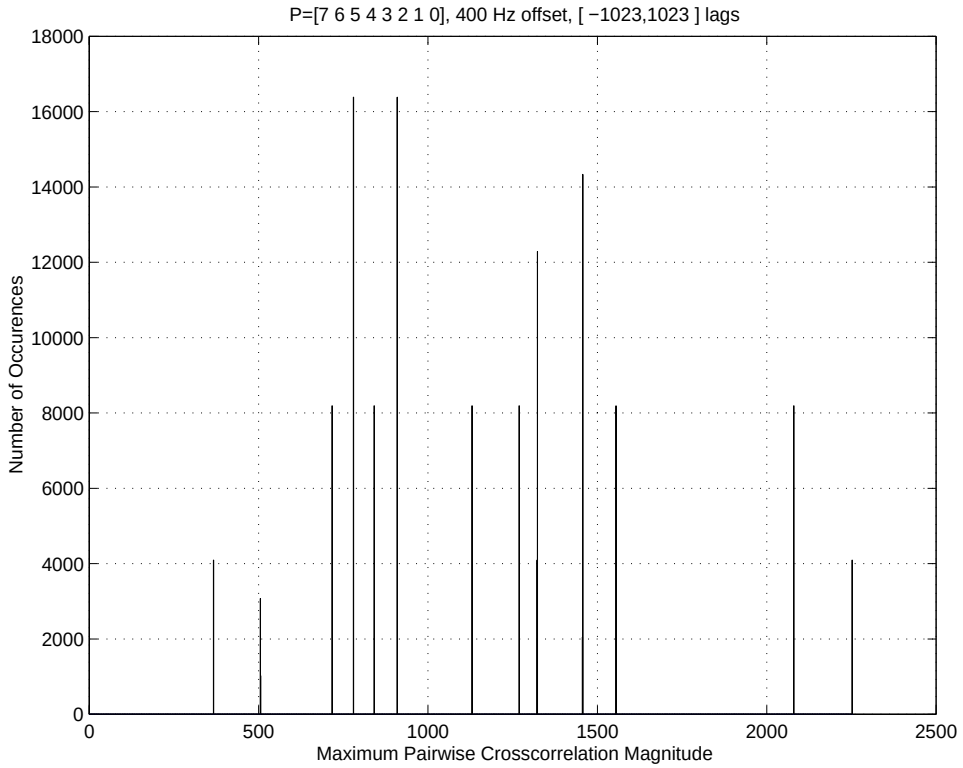

Figure 7: Crosscorrelation for 1023 chip search window and 400 Hz frequency offset: Golay codes.

Golay Codes

Figure 8 illustrates this phenomenon for a frequency offset of 400 Hz and 21 Doppler channels ranging from -1000 Hz to 1000 Hz. Here the maximum crosscorrelation magnitude over 21 Doppler channels is plotted vs. the crosscorrelation magnitude without multiple Doppler channels, i.e., the method of the previous section, (see (3)). Searching over multiple frequencies causes the number of signatures within 20 dB of the peak to increase from 5 to 11.

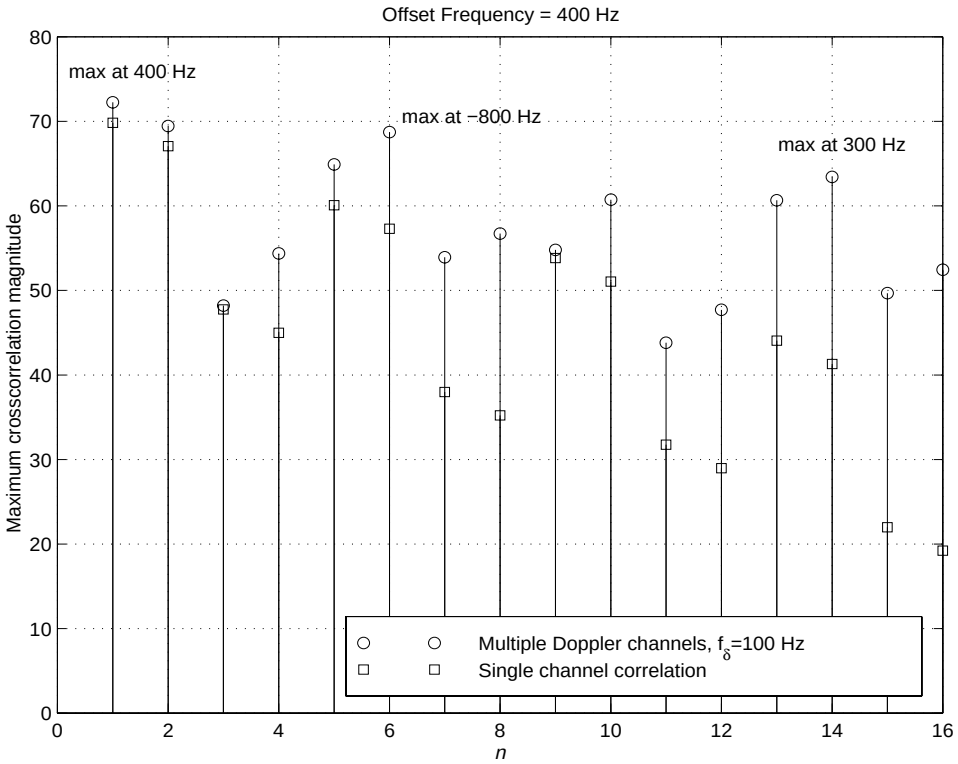

Figure 8: Golay codes: Increases in crosscorrelation due to searching over Doppler channels. Channels ranged from -1000 Hz to 1000 Hz. $f_\delta = 100$ Hz **. Frequency offset was 400 Hz.**

.

When no frequency offset is present, the effect on crosscorrelations by searching over multiple Doppler channels is even more dramatic. Figure 9 shows the maximum crosscorrelation magnitude with 0 Hz offset. Without multiple Doppler channels, there is no crosscorrelation between signatures. When the frequency axis is searched however, two signatures have correlations only 6 dB down from that of the transmitted signature. Similar results are presented in [4].

Gold Codes

The effect of searching across Doppler channels when Gold codes are used is shown in Figs. 10 and 11. Comparing these with Figs. 8 and 9 we see larger increases in correlation with Golay codes at 400 Hz offset and an uneven distribution of correlations relative to Gold codes at 0 Hz. Overall the Gold codes seem to be somewhat less sensitive to searching over Doppler channels.

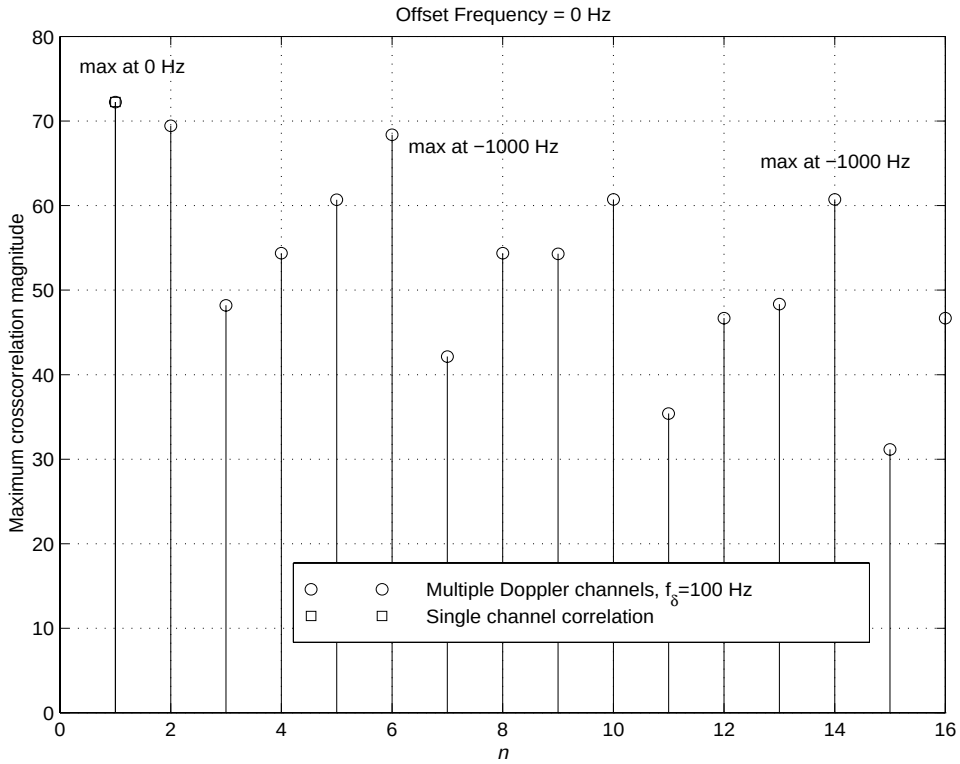

Figure 9: Golay codes: Crosscorrelation when searching over multiple Doppler channels when no frequency offset is present. Channels ranged from -1000 Hz to 1000 Hz. $f_\delta = 100$ Hz **.**

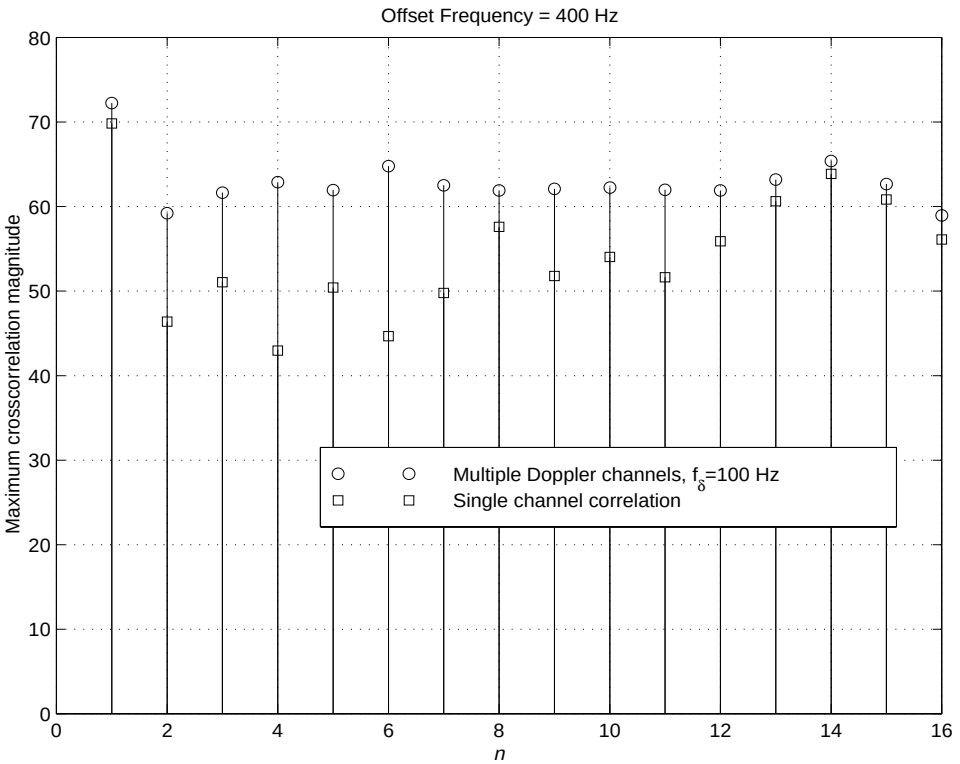

Figure 10: Gold Codes: Increases in crosscorrelation due to searching over Doppler channels ranging from -1000 Hz to 1000 Hz. $f_\delta = 100$ Hz **. Frequency offset: 400 Hz**

Similar reasoning to what was presented in the previous section infers that correlations between random codes would degrade little with this type of processing.

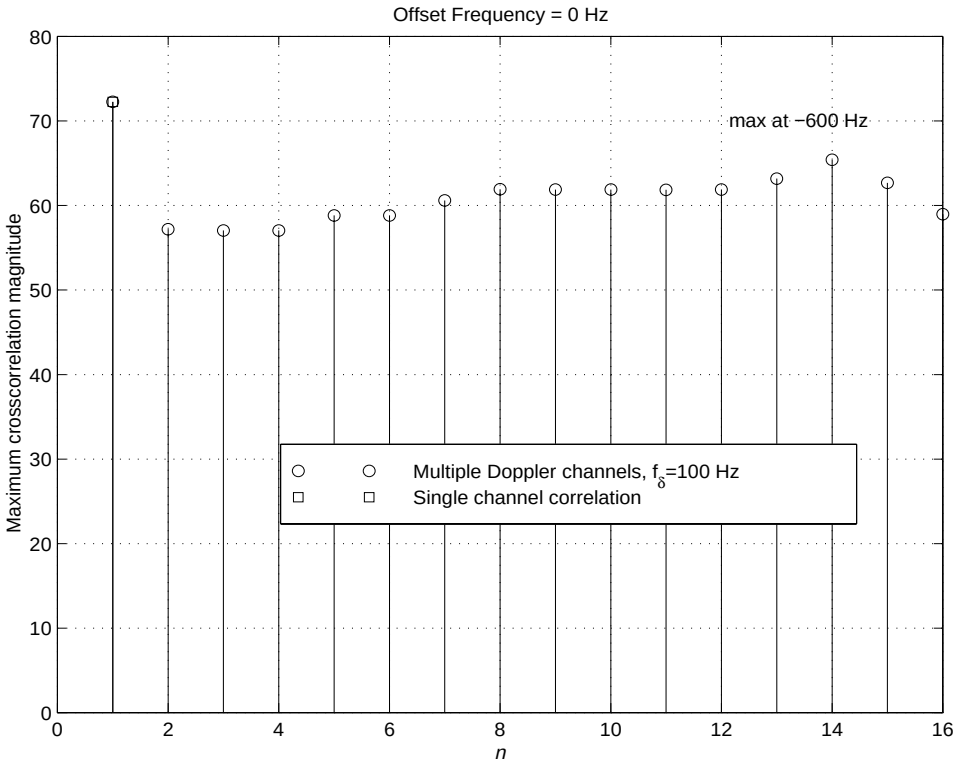

Figure 11: Gold Codes: Increases in crosscorrelation due to searching over Doppler channels ranging from -1000 Hz to 1000 Hz. $f_\delta = 100$ Hz **. Frequency offset: 400 Hz**

5.0 Offset Frequency Estimation

Besides increasing correlation between signature sequences, a frequency offset between the received preamble and the base station oscillator can degrade coherent demodulation of the message frame. By estimating this offset from the preamble, the receiver oscillator's frequency may be adjusted prior to message detection or the offset may be used as an initial condition for an automatic frequency control circuit. This is easily accomplished if the method described in the previous section is used for detection. The frequency corresponding to the bin with the largest energy can be used as an estimate [6]. The increases in crosscorrelation which occurs with this approach however may increase the false alarm rate to unacceptable limits. A second method which is relatively easy to implement is based on calculating phase differences between consecutive samples [6]. Filtering or phase unwrapping can then be applied to yield the offset frequency estimate.

The structure of the signature sequences however makes this relatively simple approach vulnerable to multiple-access interference from other RACH preambles. As an example consider the case of an interfering preamble with no offset which arrives with the same offset as the preamble whose offset frequency is to be estimated. Let s_k and $\bar{s}_k$ be the desired and interfering symbols respectively at time k and let θ be the change in phase between symbols corresponding to the desired offset frequency. Neglecting additive noise, the received signal is then

$$r_k = s_k e^{jk\theta} + \bar{s}_k . \tag{6}$$

The phase can be estimated by taking the argument of the filtered differences over $N = 16$ symbols:

$$\hat{\theta} = \arg z \tag{7}$$

$$\begin{aligned} z &= \sum_{k=1}^{N-1} s_k^* s_{k-1} r_{k-1}^* r_k \\ &= \sum_{k=1}^{N-1} e^{j\theta} + \bar{s}_k s_k^* e^{j(k-1)\theta} + s_{k-1}\bar{s}_{k-1}^* e^{jk\theta} + s_k^* \bar{s}_k s_{k-1} \bar{s}_{k-1}^* \end{aligned} \tag{8}$$

The term $s_k^* s_{k-1} \bar{s}_k \bar{s}_{k-1}^*$ in the above is a correlation between the sequence, $s_k s_{k-1}$, which comes from taking consecutive products of the desired preamble and the sequence, $\bar{s}_k s_{k-1}^*$, which comes from consecutive products of the interfering preamble. The problem comes from the fact that while cross correlations between signatures are designed to be zero, the crosscorrelations between these new sequences are generally not zero. Consequently, the last term in (8) could cause a significant bias in the estimate. This is indeed the case for the desired and interfering signatures corresponding to signatures 0 and 2 respectively from Table 1 of [1]. In this case, the last interfering term has equal magnitude to the term containing the phase information. Figure 12 plots offset frequency error standard deviation vs. interfering power for this example. Degradation begins at -3 dB reaching over 1 kHz when the interfering and desired signal power are equal
.

Gold Codes

Figure 13 indicates a similar situation to the above when Gold codes are used. Offset frequency error increases significantly once interfering power approaches desired power.

With random codes, interference will be independent between symbols and therefore the large biases shown above would not accumulate.

6.0 Conclusion

The present Gold and proposed Golay codes, while lending themselves to efficient matched filtering and, in the case of Golay codes, having excellent autocorrelation properties, suffer from several problems:

- Large crosscorrelations (>.5) between signatures at offsets of more than 255 chips.
- Degraded crosscorrelation properties in channels with frequency offset
- Large multiple access interference when phase differences are used for frequency acquisition.

Because random codes seem to be immune to these problem, Nokia's proposal of using long srambling codes, which approximate random codes, should be carefully evaluated. The degree to

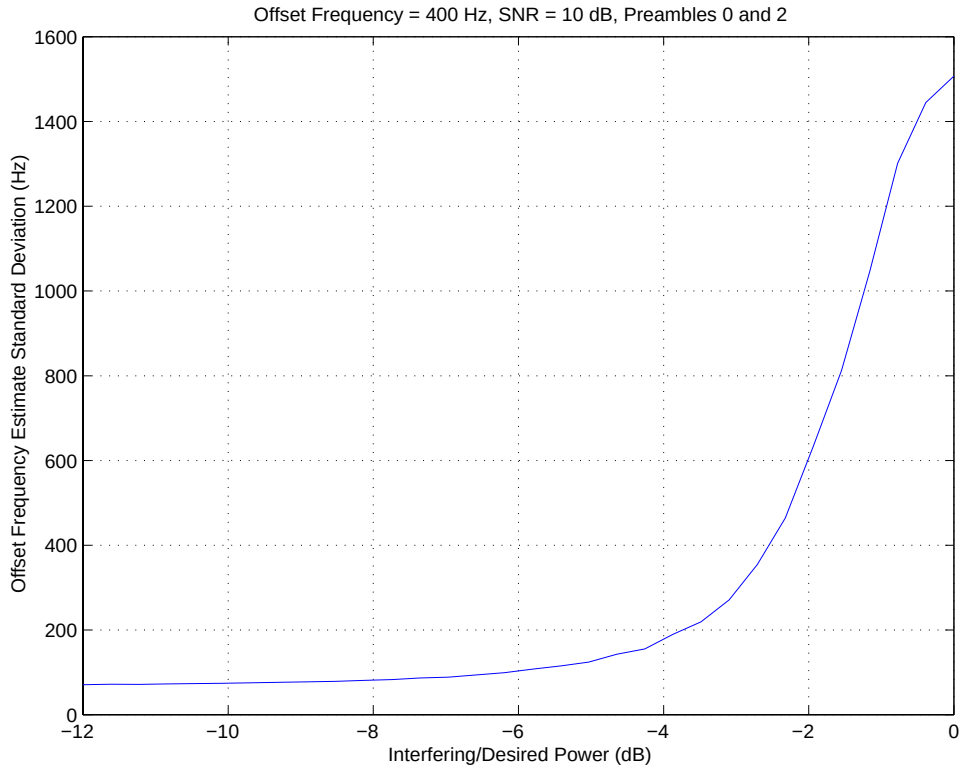

Figure 12: Effect of interfering preamble on offset frequency estimation: Golay Codes

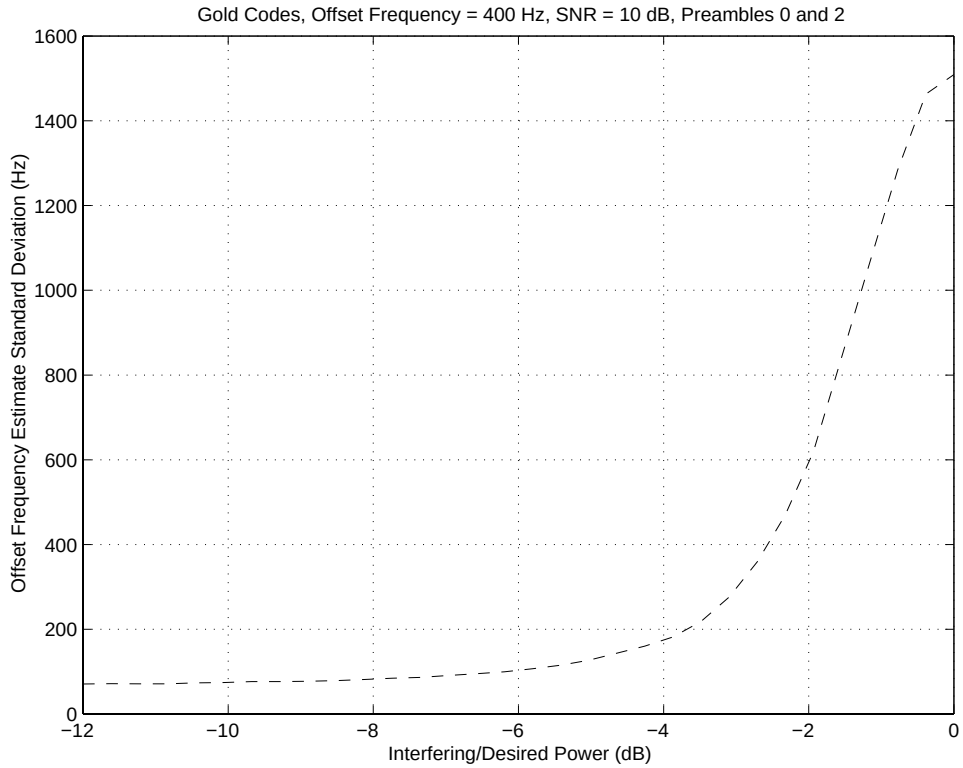

Figure 13: Effect of interfering preamble on offset frequency estimation: Gold Codes

with the above issues are tied to complexity constraints should also be studied.

7.0 References

[1] *New RACH Preambles with Low Auto-correlation Sidelobes and Reduced Detector Complexity*, TSGR1#3(99)205, Ericsson.

[2] *Comments on the Proposed RACH Sequence Structure*, TSGR1#4(99)377., Nokia

[3] *Random Access Preamble Detection in Presence of Doppler*, Tdoc SMG2 UMTS L1 620/98, Interdigital Comm. Corp.

[4] *Effect of Doppler on Current and Proposed RACH Preambles*, TSGR1#4(99)336, Interdigital Comm. Corp.

[5] *Preamble Detection in Doppler Spread Using Multiple Doppler Channel*s, TSGR1#4(99)403, Interdigital Comm. Corp.

[6] H. Meyr et al, Digital Communication Receivers, ch. 8, New York: Wiley Interscience., 1998.