Title: Correlation Computation of SCM

File: SCM-096-LUC-Correlation-Computation

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1 Introduction

We compute the correlation of SCM [1] and give a series expression for cross or auto correlation as a function of parameters like antenna spacing, angle of arrival etc. Since such a representation is exact i.e. when infinite oscillators are present (instead of few as in most practical implementations), it provides a benchmark to test the various implementations like the ones mentioned in [2, 3, 4]. We plot the correlation as a function of distance for some cases of interest.

2 Derivation of Correlation in a Wide-Band SCM

Let us consider the correlation between two elements that have the following parameters:

d_b^i	is the antenna spacing at the BS (Node B) for element i ($i = 1, 2$)
Δd_b	$= d_b^1 - d_b^2$
d_m^i	is the antenna spacing at the MS (UE) for element i ($i = 1, 2$)
Δd_m	$= d_m^1 - d_m^2$
θ	is the angle w.r.t. BS broadside to a given sub-path
$\bar{\theta}$	is the average of θ taken over all sub-paths
β	is the angle w.r.t. UE broadside to a given sub-path
$\bar{\beta}$	is the average of β taken over all sub-paths
ϕ_v	is the angle of the speed vector with respect to the MS array's broadside
$k = \frac{2\pi}{\lambda}$	is the wave number, where λ is the wave-length
r^i	is the distance or (lag) in meters for element i (i = 1,2)
Δr	is the distance lag between the two elements ($= r^1 - r^2$)
$\alpha(\theta, \beta)$	is the angle associated with a given sub-path and is uniformly distributed in $[0, 2\pi]$
$L(\theta, \bar{\theta}, \sigma_b)$	is the Laplacian spread at BS
$L(\beta, \bar{\beta}, \sigma_m)$	is the Laplacian spread at MS

where

$$L(\theta, \bar{\theta}, \sigma_b) = \frac{1}{\sigma_b\sqrt{2}} \exp\left(-\frac{\sqrt{2} \mid \theta - \bar{\theta} \mid}{\sigma_b}\right),$$

and $L(\beta, \bar{\beta}, \sigma_m)$ is defined similarly. Note that $\bar{\theta}$ and $\bar{\beta}$ can be related to the angle quantities defined in [1].

We will not consider antenna patterns for the series representation. They can be added easily though the series expression will become messier. The ray for the chosen sub-path for the ith element is given as

$$h_i(r^i) = \int_{-\pi+\bar{\theta}}^{\pi+\bar{\theta}} \int_{-\pi+\bar{\beta}}^{\pi+\bar{\beta}} e^{jkd_b^i \sin(\theta)+jkd_m^i \sin(\beta)+jkr^i \cos(\beta-\phi_v)+j\alpha(\theta,\beta)} L(\theta, \bar{\theta}, \sigma_b) L(\beta, \bar{\beta}, \sigma_m) d\theta d\beta. \quad (1)$$

Since $E\{e^{j\alpha(\theta,\beta)-j\alpha(\acute{\theta},\acute{\beta})}\} = \delta(\theta - \acute{\theta})\delta(\beta - \acute{\beta})$, where $\delta(\cdot)$ is a Dirac-delta function, hence

$$\rho \;=\; E\{h_1(r^1)h_1^*(r^2)\}$$

$$
\begin{aligned}
&= \int_{-\pi+\bar\theta}^{\pi+\bar\theta}\int_{-\pi+\bar\beta}^{\pi+\bar\beta} e^{jk\Delta d_b\sin(\theta)+jk\Delta d_m\sin(\beta)+jk\Delta r\cos(\beta-\phi_v)} L(\theta,\bar\theta,\sigma_b)L(\beta,\bar\beta,\sigma_m)d\theta d\beta \\
&= \int_{-\pi+\bar\theta}^{\pi+\bar\theta} e^{jk\Delta d_b\sin(\theta)} L(\theta,\bar\theta,\sigma_b)d\theta \int_{-\pi+\bar\beta}^{\pi+\bar\beta} e^{jk\Delta d_m\sin(\beta)+jk\Delta r\cos(\beta-\phi_v)} L(\beta,\bar\beta,\sigma_m)d\beta \\
&\overset{1}{=} \int_{-\pi+\bar\theta}^{\pi+\bar\theta} e^{jk\Delta d_b\sin(\theta)} L(\theta,\bar\theta,\sigma_b)d\theta \int_{-\pi+\bar\beta}^{\pi+\bar\beta} e^{jkc\sin(\beta+\gamma)} L(\beta,\bar\beta,\sigma_m)d\beta \\
&= \int_{-\pi+\bar\theta}^{\pi+\bar\theta} e^{jk\Delta d_b sin(\theta)} L(\theta,\bar\theta,\sigma_b)d\theta \int_{-\pi+\bar\beta+\gamma}^{\pi+\bar\beta+\gamma} e^{jkc\sin(\beta)} L(\beta,\bar\beta+\gamma,\sigma_m)d\beta, \\
&= Z(\bar\theta,k\Delta d_b,\sigma_b)Z(\bar\beta+\gamma,kc,\sigma_m) \qquad (2)
\end{aligned}
$$

where in '1', $c=\sqrt{(\Delta d_m)^2+(\Delta r)^2+2\Delta d_m\Delta r\sin(\phi_v)}$, $\gamma=\tan^{-1}\left(\frac{\Delta r\cos(\phi_v)}{\Delta d_m+\Delta r\sin(\phi_v)}\right)$, and

$$
\begin{aligned}
Z(\bar\theta,C,\sigma) &= \int_{-\pi+\bar\theta}^{\pi+\bar\theta} e^{jC\sin(\theta)} L(\theta,\bar\theta,\sigma)d\theta \\
\sigma\sqrt{2}Z(\bar\theta,C,\sigma) &= \int_{-\pi+\bar\theta}^{\pi+\bar\theta} e^{jC\sin(\theta)} e^{\frac{-\sqrt{2}|\theta-\bar\theta|}{\sigma}} d\theta \\
&= e^{-\frac{\sqrt{2}\bar\theta}{\sigma}}\int_{-\pi+\bar\theta}^{\bar\theta} e^{jC\sin(\theta)}e^{\frac{\sqrt{2}\theta}{\sigma}}d\theta + e^{\frac{\sqrt{2}\bar\theta}{\sigma}}\int_{\bar\theta}^{\pi+\bar\theta} e^{jC\sin(\theta)}e^{\frac{-\sqrt{2}\theta}{\sigma}}d\theta \\
&= e^{-\frac{\sqrt{2}\bar\theta}{\sigma}} Y\left(-\pi+\bar\theta,\bar\theta,C,\frac{\sqrt{2}}{\sigma}\right) + e^{\frac{\sqrt{2}\bar\theta}{\sigma}} Y\left(\bar\theta,\pi+\bar\theta,C,\frac{-\sqrt{2}}{\sigma}\right), \qquad (3)
\end{aligned}
$$

where $Y(a,b,C,D)=\int_a^b e^{jC\sin(\theta)}e^{D\theta}d\theta$. Let

$$Y(a,b,C,D)=R(a,b,C,D)+jI(a,b,C,D), \qquad (4)$$

where R and I are real valued functions. Hence

$$
\begin{aligned}
R(a,b,C,D) &= \text{real}\left(\int_a^b e^{jC\sin(\theta)}e^{D\theta}d\theta\right) \\
&= \sum_{n=0}^{\infty}\frac{(-C^2)^n}{(2n)!}\int_a^b \sin^{2n}(\theta)e^{D\theta}d\theta \\
&= \sum_{n=0}^{\infty}\frac{(-C^2)^n}{(2n)!}\left(\frac{1}{2^{2n}}\binom{2n}{n}\int_a^b e^{D\theta}d\theta + \sum_{k=1}^{n}T_1(k,n)\int_a^b\cos(2k\theta)e^{D\theta}d\theta\right) \\
&= \sum_{n=0}^{\infty}\frac{(-C^2)^n}{(2n)!}\left(\frac{e^{Db}-e^{Da}}{2^{2n}D}\binom{2n}{n} + \right. \\
&\qquad \left. \sum_{k=1}^{n}\frac{T_1(k,n)\Big(e^{D\theta}(D\cos(2k\theta)+2k\sin(2k\theta))\Big)\Big|_a^b}{(2k)^2+D^2}\right) \\
&= \frac{e^{Db}-e^{Da}}{D}J_0(C) + \sum_{n=0}^{\infty}\sum_{k=1}^{n}\frac{T_1(k,n)\Big(e^{D\theta}(D\cos(2k\theta)+2k\sin(2k\theta))\Big)\Big|_a^b}{(2k)^2+D^2}, \qquad (5)
\end{aligned}
$$

where $T_1(k,n) = \frac{(-1)^{2n-k}}{2^{2n-1}}\binom{2n}{n-k}$ for $1 \le k \le n$, $J_0(\cdot)$ is the Bessel function and

$$\begin{aligned} I(a,b,C,D) &= \text{imag}\left(\int_a^b e^{jC\sin(\theta)}e^{D\theta}d\theta\right) \\ &= \sum_{n=0}^{\infty}\frac{C(-C^2)^n}{(2n+1)!}\int_a^b \sin^{2n+1}(\theta)e^{D\theta}d\theta \\ &= \sum_{n=0}^{\infty}\frac{C(-C^2)^n}{(2n+1)!}\sum_{k=0}^{n}T_2(k,n)\int_a^b \sin((2k+1)\theta)e^{D\theta}d\theta \\ &= \sum_{n=0}^{\infty}\frac{C(-C^2)^n}{(2n+1)!}\sum_{k=0}^{n}\frac{T_2(k,n)}{(2k+1)^2+D^2} \\ &\qquad \left(e^{D\theta}(D\sin((2k+1)\theta)-(2k+1)\cos((2k+1)\theta))\right)\Big|_a^b, \end{aligned} \tag{6}$$

where $T_2(k,n) = \frac{(-1)^{2n-k}}{4^n}\binom{2n+1}{n-k}$ for $0 \le k \le n$. Substituting the expressions for T_1 and T_2 into Eq. (5) and (5) respectively and simplifying, we get

$$R(a,b,C,D) = \frac{e^{Db}-e^{Da}}{D}J_0(C) + 2\sum_{k=1}^{\infty}J_{2k}(C)\frac{\left(e^{D\theta}(D\cos(2k\theta)+2k\sin(2k\theta))\right)\Big|_a^b}{(2k)^2+D^2} \tag{7}$$

and

$$I(a,b,C,D) = 2\sum_{k=0}^{\infty}J_{2k+1}(C)\frac{\left(e^{D\theta}(D\sin((2k+1)\theta)-(2k+1)\cos((2k+1)\theta))\right)\Big|_a^b}{(2k+1)^2+D^2}. \tag{8}$$

Substituting the series representation of R and I into Eq. (4) and in Eqs. (3) & (2), we get the series representation of the correlation ρ.

If we call the summation as a perturbation in Eqs. (7) and (8), then it is clear that the effect of angle spread on the correlation is to scale the Bessel function (which is the correlation for uniform angle spread) and to perturb it.

3 Comparison of Correlation

We plot the auto-correlation function that we obtain by different calculations in Figs. (1), (2) and (3). In the first calculation, we plot the auto-correlation that is calculated after generating fades from 20 sub-rays SCM having equal power and non-uniform angle spacing with $\sigma_m = 35^o$. In the second calculation, we plot the auto-correlation that is obtained by numerical integration of Eq. (2). In the third calculation, the auto-correlation function is computed by series expansion as given in Section (2). For comparison purposes, we also plot the Bessel function. As can be seen from Figs. (1), (2) and (3) that plot the auto-correlation for various mobile directions relative to angle of arrival, the numerical integration and series expansion match quite closely to each other. There is a small difference between the SCM model with 20 sub-rays and the one given by the integral. This difference is expected since the integral assumes infinite number of sub-rays as opposed to finite as in SCM model.

The normalized cross-correlation function is plotted in Figs. (4), (5) and (6) for different mobile directions. The angle of arrival is always chosen to be 0^o for these plots. For the case of perpendicular mobile direction relative to the ray arrival, the imaginary part is zero as in the case of auto-correlation. The SCM 20 sub-ray model matches closely with the integral. The antenna spacings for all the simulations for cross-correlations were done with $\Delta d_b = \lambda$ and $\Delta d_m = \lambda/2$. In Fig. (7), we plot the normalized cross-correlation with $\Delta d_b = 10\lambda$ and $\Delta d_m = \lambda/2$. Due to larger antenna spacing at the BS, the correlation becomes quite small in this case.

To check the correlation for larger time delay, we plot the auto-correlation and cross-correlation for distance of up to 20λ in Figs. (8) and (9). In both the cases, the mobile direction is perpendicular to the ray arrival. As can be seen from these figures, for larger distance the correlation obtained by simulation of 20 sub-ray SCM becomes larger and doesn't match the numerical computation of the integral, which it did for smaller distances. This is a limitation of finite (20 for the present case) sub-rays based model that the correlation doesn't decay as it should. This indicates that care must be taken in using this model in a simulation. If the encoded block (with or without combining of re-transmitted sub-packets) exceeds the distance where correlation doesn't match the expected, then it may be better to use SCM with more sub-rays instead of 20.

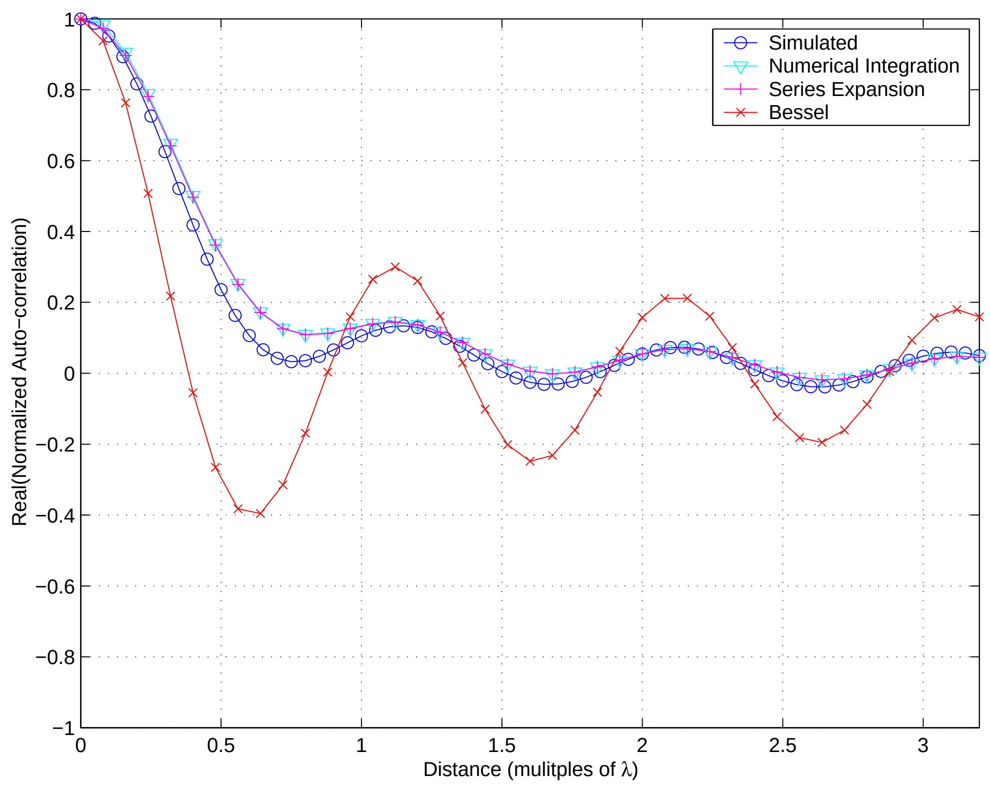

Figure 1: Normalized Auto-correlation function as a function of distance (in multiples of wavelength) with mobile direction perpendicular to the ray arrival.

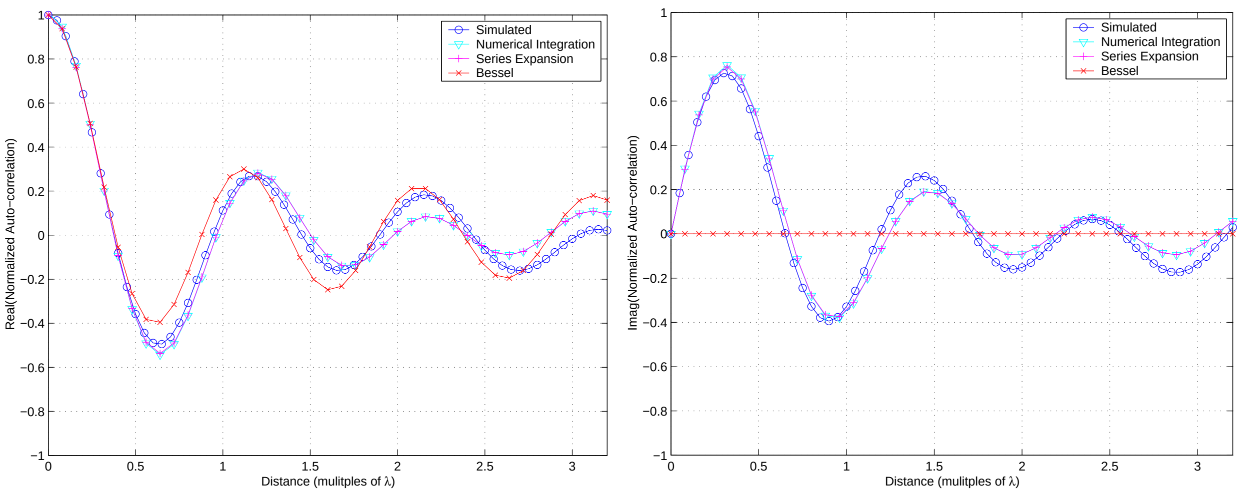

Figure 2: Normalized Auto-correlation function as a function of distance (in multiples of wavelength) with mobile direction 45^o to the ray arrival.

4 Conclusions

In conclusion, we provided the computation of correlation between same or distinct rays in integral form. This integral form is also given as a series expression. This serves as a benchmark where different schemes with alternate fading generation methods can be compared against. The 20 sub-ray SCM agrees closely with the correlation integral (that assumes infinite or continuum of rays) when the distance is small (about 5λ). For larger distances, the correlation obtained by the 20 sub-ray model doesn't decay as it should, and becomes larger because of the limitation of finite sub-rays.

References

[1] SCM-062-SCM-Text v.1.9, SCM Conference Call, October 10, 2002.

[2] Lucent, System model verification and calibration, SCM-069R2, SCM Conference Call, Nov 21, 2002.

[3] Nortel Networks, SCM-065_v2, SCM Model Correlations, Teleconference, Oct 10, 2002.

[4] Motorola, SCM-086SCM-Fading-Models, SCM Conference Call, Dec 19, 2002.

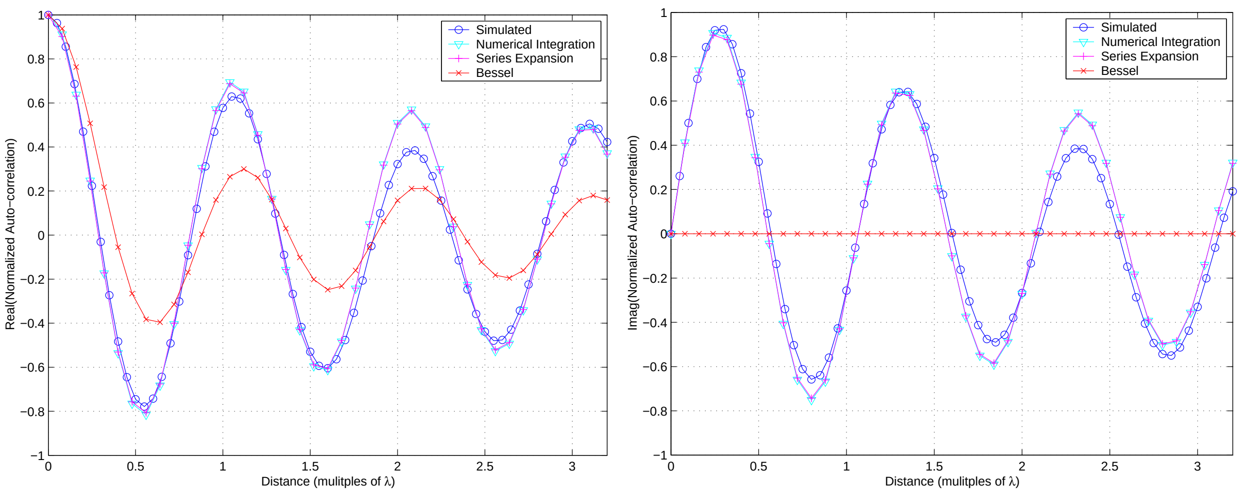

Figure 3: Normalized Auto-correlation function as a function of distance (in multiples of wavelength) with mobile direction towards ray arrival.

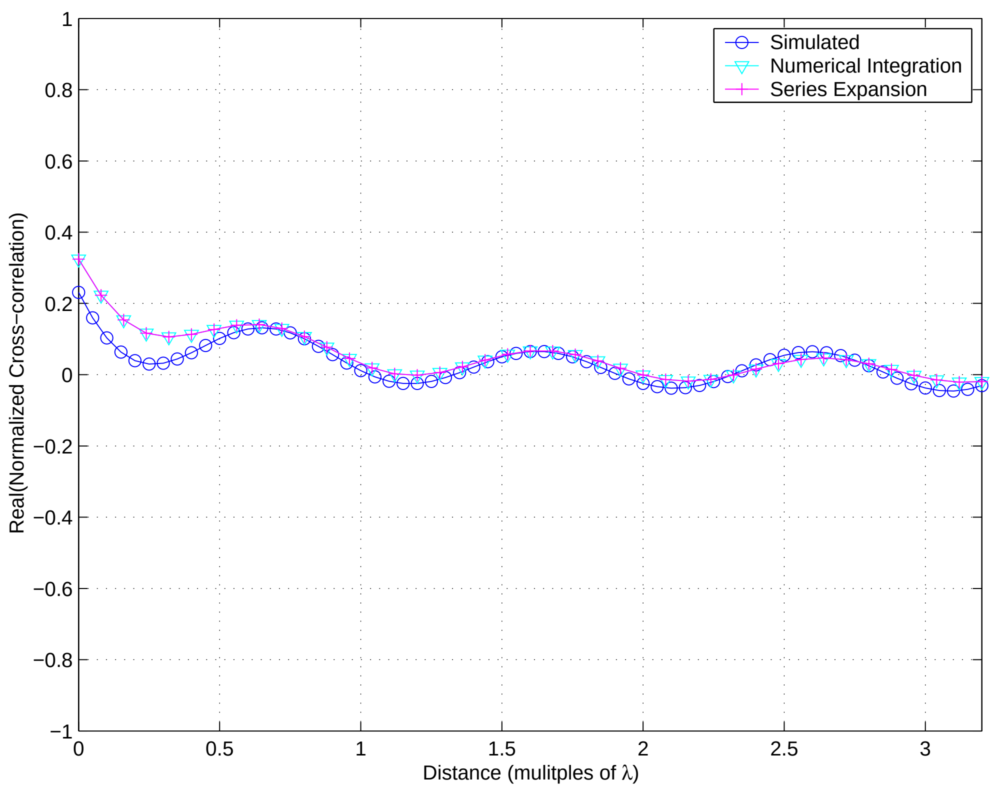

Figure 4: Normalized Cross-correlation function as a function of distance (in multiples of wavelength) with mobile direction perpendicular to the ray arrival with $\Delta d_b = \lambda$ and $\Delta d_m = \lambda/2$.

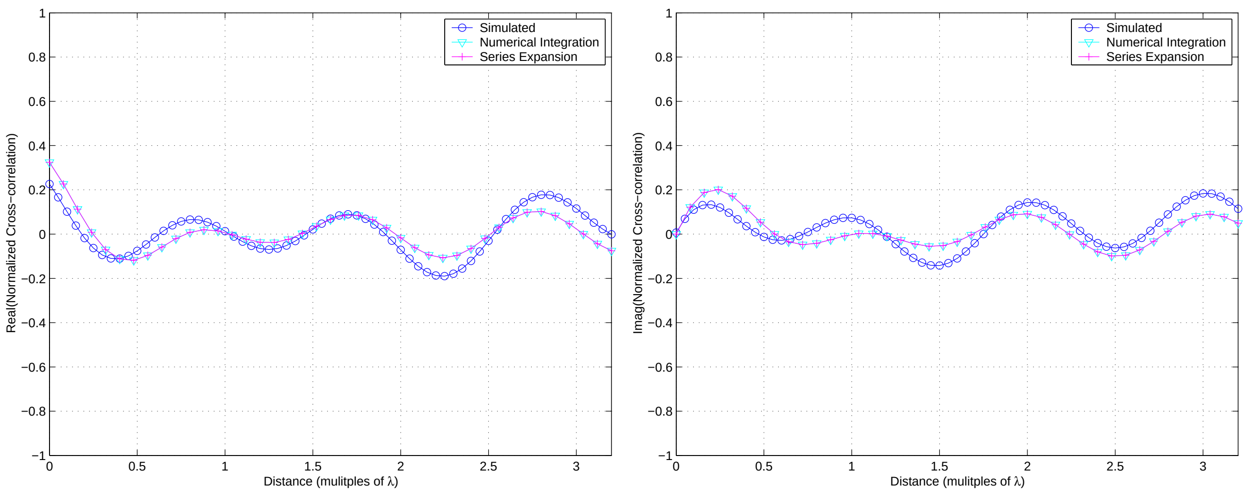

Figure 5: Normalized Cross-correlation function as a function of distance (in multiples of wavelength) with mobile direction 45^o to the ray arrival with $\Delta d_b = \lambda$ and $\Delta d_m = \lambda/2$.

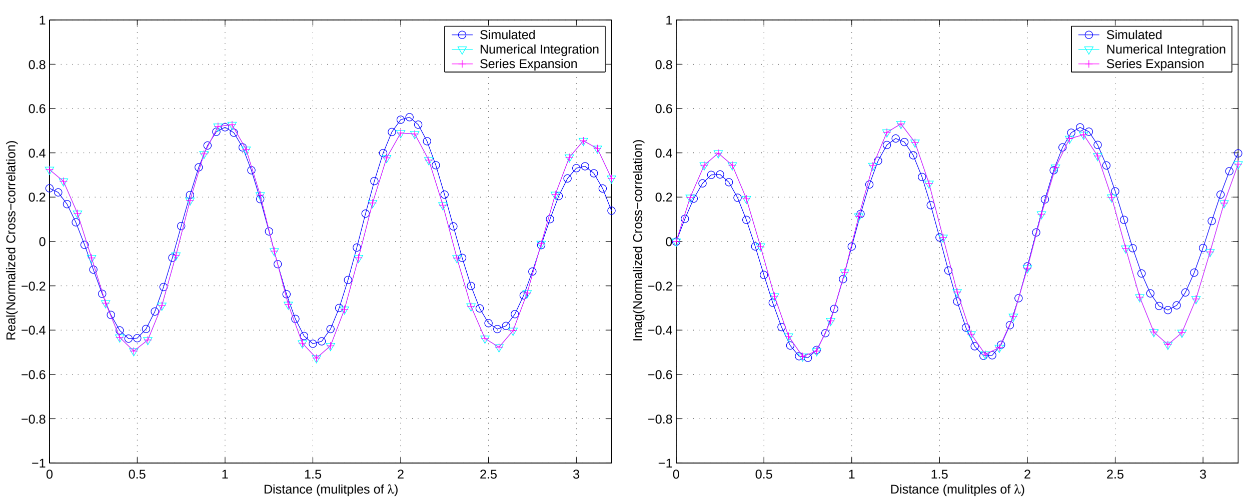

Figure 6: Normalized Cross-correlation function as a function of distance (in multiples of wavelength) with mobile direction towards ray arrival with $\Delta d_b = \lambda$ and $\Delta d_m = \lambda/2$.

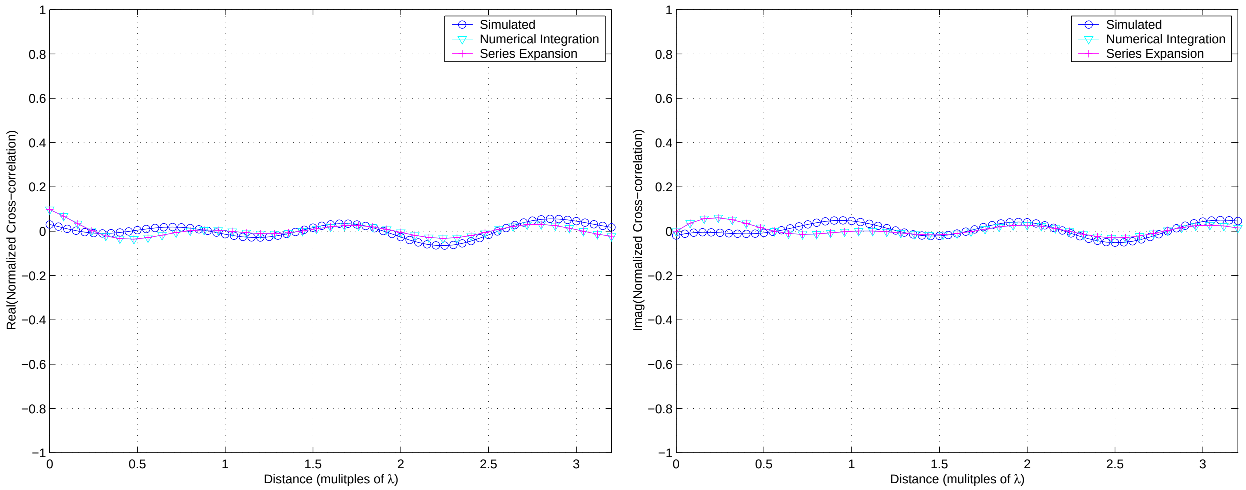

Figure 7: Normalized Cross-correlation function as a function of distance (in multiples of wavelength) with mobile direction 45^o to the ray arrival with $\Delta d_b = 10\lambda$ and $\Delta d_m = \lambda/2$.

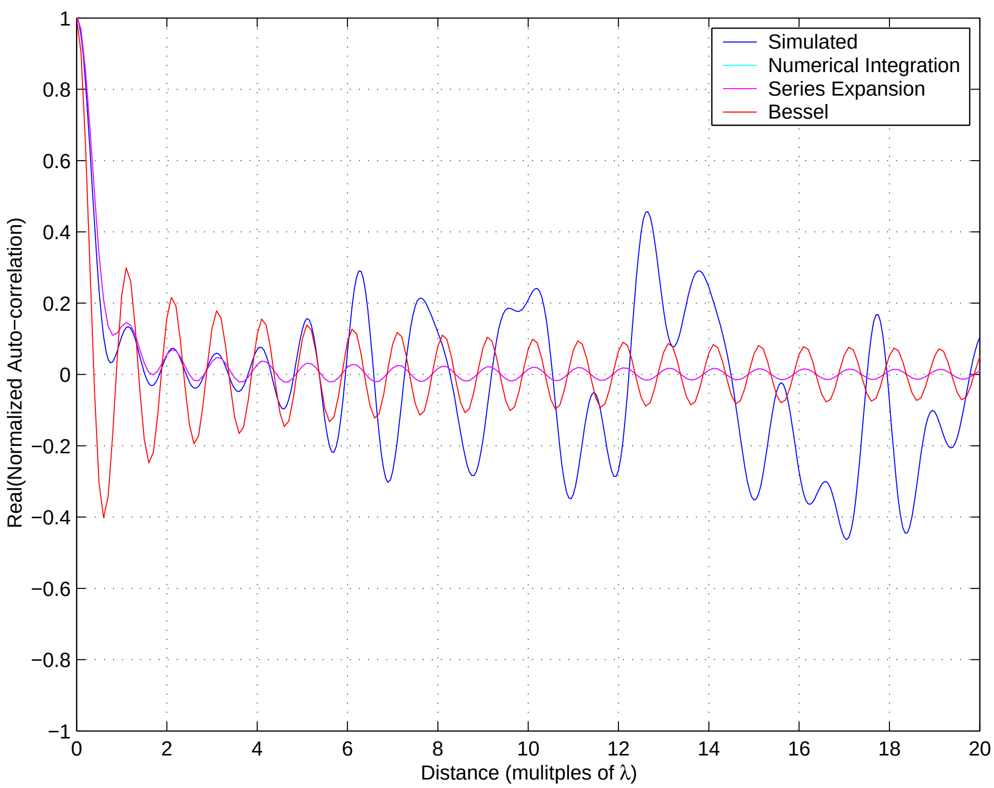

Figure 8: Normalized Auto-correlation function as a function of distance (in multiples of wavelength) with mobile direction perpendicular to the ray arrival.

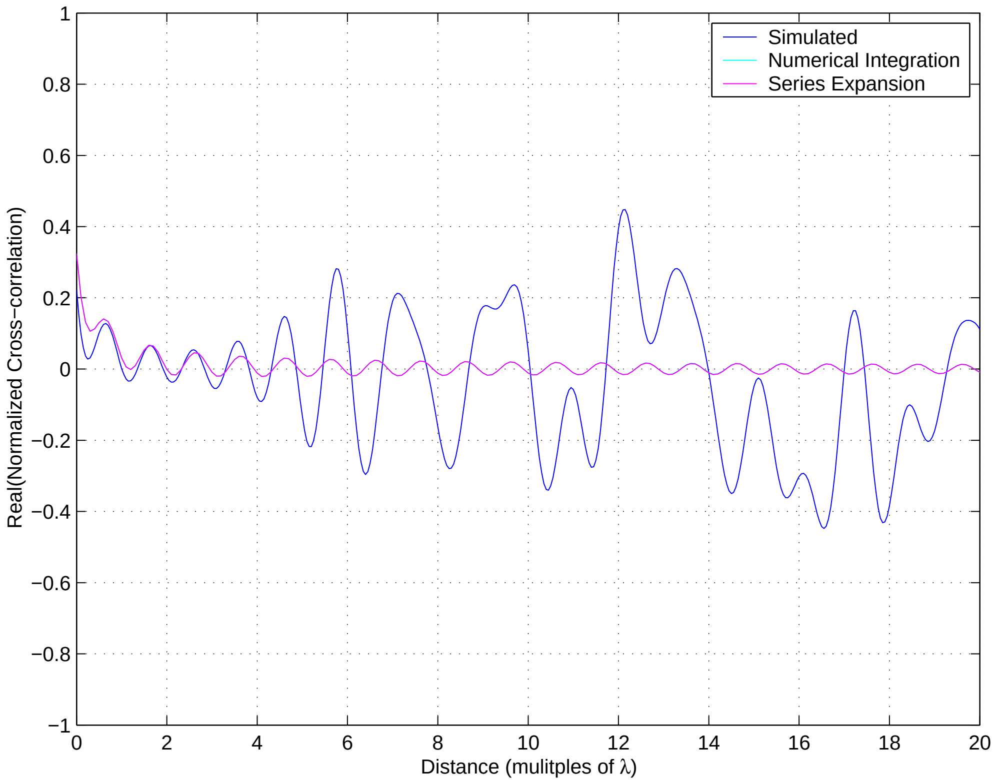

Figure 9: Normalized Cross-correlation function as a function of distance (in multiples of wavelength) with mobile direction perpendicular to the ray arrival.