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Source: **Broadcom Corporation**

Title: **ZC Based P-SCH Sequences and Initial Frequency Estimation Uncertainty**

Agenda item: **5.2**

Document for: **Discussion/Information**

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1. Introduction

This document addresses a problem that exists with ZC sequences when initial coarse frequency estimation/correction is performed based on the P-SCH when these sequences are used for primary synchronization. Based on the currently adopted system architecture employing a narrowband SCH occupying the centre 64 sub-carriers of an OFDM signal (including the DC sub-carrier), systems would typically need to make use of low-pass filtering and decimation to arrive at a time-domain signal that will allow for the smallest and most efficient time domain primary synchronization matched filter. It is shown that certain ZC sequences have associated with them a frequency estimation inaccuracy which is strongly dependent upon the decimation/down-sampling phase used when decimating/down-sampling the low-pass filter output. The frequency offset estimation inaccuracy problem related to ZC sequences has also been discussed in [4].

This problem is considered for the following sequences and scenarios:

- Length 64 ZC sequences.
- Constant Amplitude Random sequences. The construction of this sequence type will be shown and in general this type of sequence seems to suffer less from this sensitivity to down-sampling phase.
- Frequency Domain Interleaved ZC sequences [1]. It is shown that with careful selection, frequency domain interleaved ZC sequences can be found that are less susceptible to this phenomenon. Those interleaved ZC sequences that are less susceptible to this seem to be similar in nature to the Constant Amplitude Random sequences which are considered.
- The effect that sequence element puncturing and sub-carrier assignment has on the initial frequency estimation accuracy problem is also addressed.
- Full-rate matched filtering is also considered as a possible means to overcoming the identified phenomenon. However, it is found that when using a full-rate approach, the problem manifests itself in a different manner where frequency offset inaccuracy is associated with timing relative to ideal rather than a down-sampling phase which is no longer relevant in this case.

On the surface of it, this document may seem to address implementation/architectural issues however it rather aims at emphasizing the fact that fundamental properties of some of the sequences considered are problematic, regardless of the receiver architecture that may be adopted. Sequences that would encumber the receiver architecture should obviously rather be avoided and the least burdensome sequences identified and used.

Figure 1 presents a block diagram of the problem considered, i.e. how accurate the initial frequency estimates are that are obtained when the abovementioned sequence types are used for the P-SCH and how sampling phase at the decimator output, sequence element puncturing and sub-carrier assignment affect the initial/coarse frequency estimates obtained by means of the primary synchronization sequence.

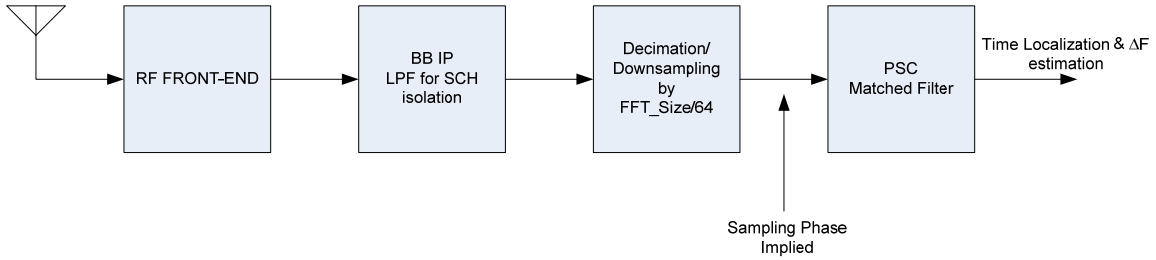


Figure 1: The problem considered: The effect of down-sampling/decimation phase on initial/coarse frequency estimation.

2. Filtering and Sampling of the SCH

Consider, for example, a M sub-carrier system with the centre N sub-carriers assigned to the SCH with a sequence of length N mapped to these carriers, including the zeroed DC sub-carrier. This implies that in order to have the least complex matched filter for identifying the P-SCH sequence, the matched filter should consist of the least possible number of samples (i.e. N samples). The filtering and down-sampling process implies that one of M/N possible phases could be chosen as the decimation/down-sampling phase.

When mapping information into the centre N sub-carriers of a M sub-carrier system only and setting to zero the remainder of the carriers(i.e. higher frequency carriers) it effectively amounts to an M carrier signal that has been perfectly brick-wall filtered to pass only the low frequency information, which then renders the low frequency information(i.e. sequence mapped to the N centre carriers) M/N times over-sampled. In the investigations undertaken a 512 carrier system with 64 carriers assigned to the SCH has been considered implying 8 possible down-sampling phases.

A graphic depiction of the process followed in arriving at an interpolated and filtered (via IFFT) replica of the P-SCH signal is presented in
Figure 2. The IFFT of the original (base rate) signal would typically be the replica against which correlation will be performed. The interpolated (over-sampled) signal can be down-sampled at any of its M/N phases in order to perform the matched filtering (correlation).

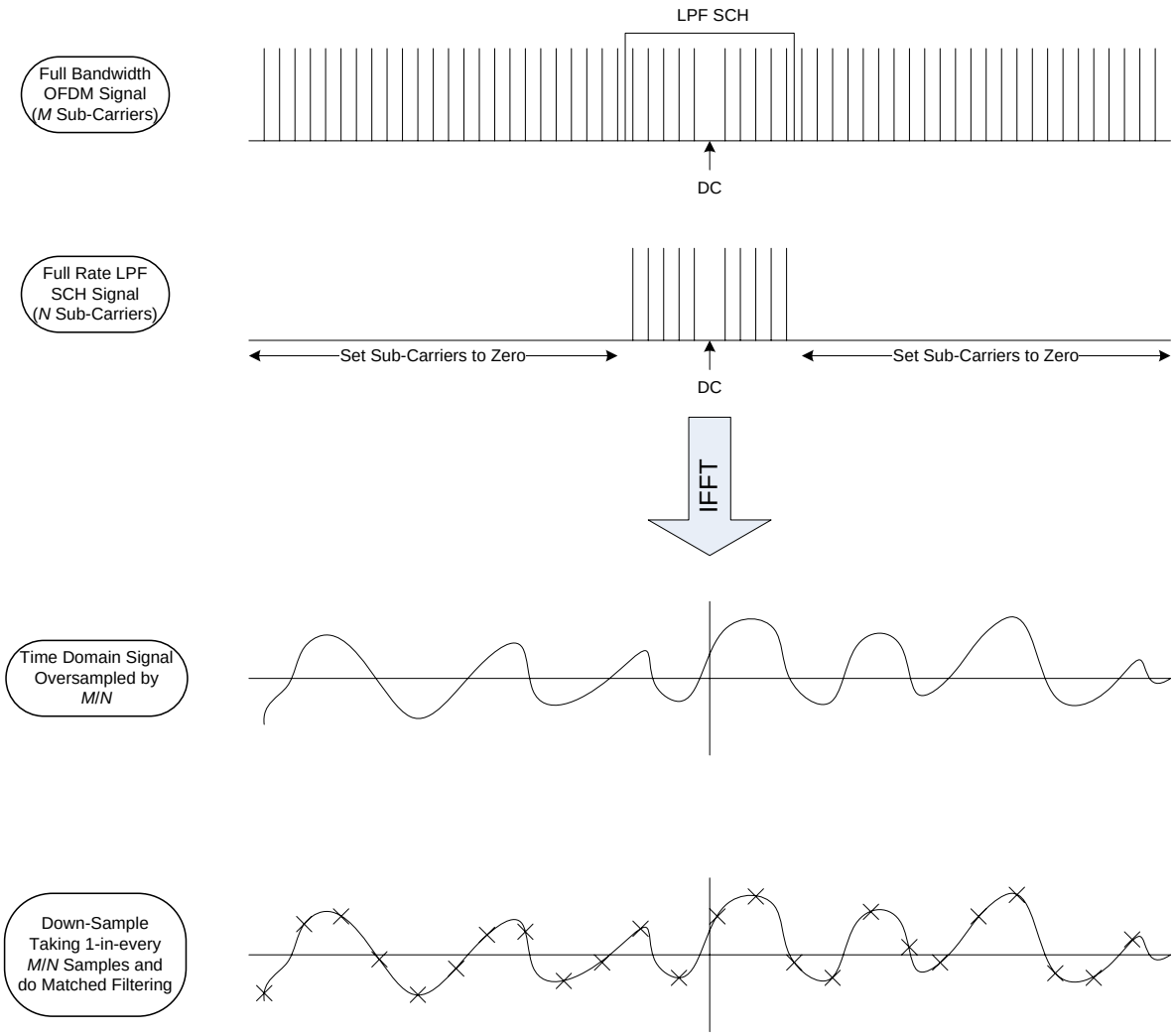
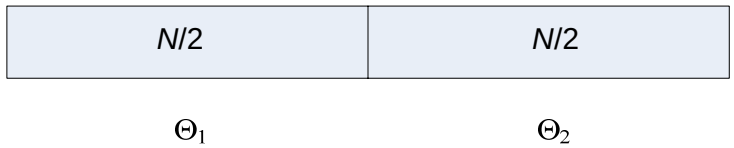


Figure 2: Graphic representation of approach followed in this investigation.

3. Partial correlation and Frequency Offset

Figure 3 shows the often employed partial correlation approach to frequency offset estimation. Based on the difference in angles observed between the first and second halves of, in this case, the P-SCH correlation over a length N sequence, the frequency offset is estimated as shown in Figure 3. F_{sc} is the sampling frequency of the partial correlation process. This is the approach followed in determining frequency offsets throughout this investigation.



$$\Delta F = \left[\frac{\Theta_2 - \Theta_1}{2\pi} \right] F_{sc}$$

Figure 3: The typical partial correlation approach to frequency offset estimation.

4. Sequence Element Puncturing & Sub-carrier Assignment

As mentioned, length 64 sequences are considered throughout and the manner in which a length 64 sequence is arrived at for assignment to 64 sub-carriers including the zeroed DC sub-carrier can be done in a multitude of ways. First, given a length 64 sequence as the starting sequence and the zeroed DC sub-carrier position which is obviously fixed, one can puncture any one of the original length 64 sequence elements and assign the remaining elements to the available 63 sub-carrier positions. This process is presented graphically in Figure 4.

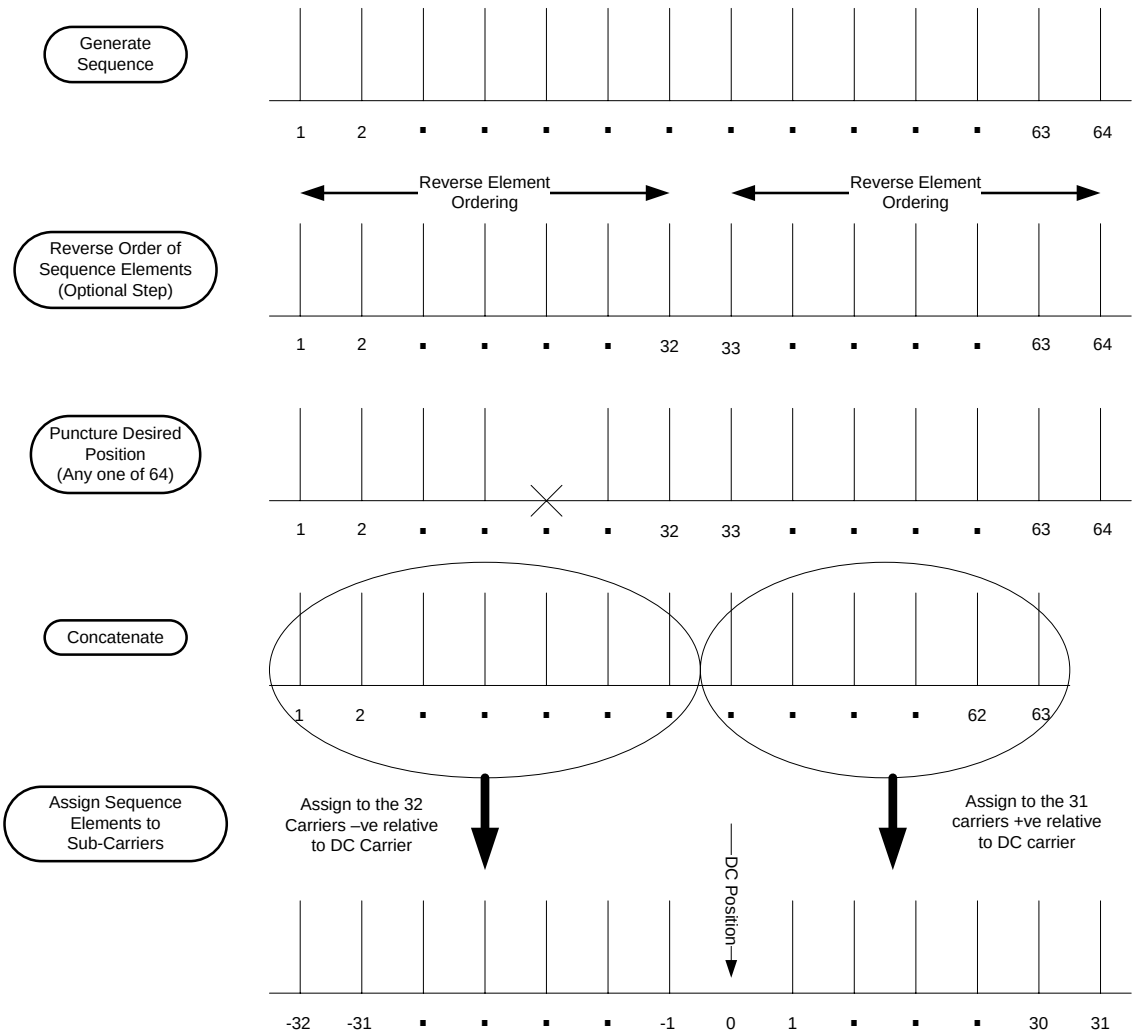


Figure 4: Sequence Generation, element ordering, element puncturing and sub-carrier assignment presented graphically.

5. Format of Results to Follow

In all of the results to follow a similar presentation of information will be used. In these figures the sequence of data presented (top to bottom) is as follows:

- Initial frequency/frequency offset estimate accuracies as a function of down-sampling/decimation phase. Frequency estimation based on the partial correlation/phase difference approach commonly used.
- Aperiodic autocorrelation as a function of the down-sampling/decimation phase.
- P-SCH time domain sequence real and imaginary components obtained via IFFT.
- P-SCH frequency domain sequence elements obtained directly from ZC sequence generation [2] and assigned to OFDM sub-carriers. The red asterisk in each of these plots represents the position in which the length 64 ZC sequence has been punctured in order to be able to assign 64 sequence elements to the 64 centre sub-carriers, including the DC sub-carrier which carries an assignment of zero. Sequence re-ordering may have been applied.
- Frequency domain magnitude of the sequence assigned to the 64 centre sub-carriers including the null DC sub-carrier.

6. RESULT: ZC Sequence Initial Frequency Estimation

As mentioned earlier the focus of this investigation is on length 64 sequences in accordance with the working assumptions from the meeting in Kobe [3]. In many of the discussions on the suitability of ZC sequences, some have been identified for their favourable correlation properties and reasonably low susceptibility to the time-shift anomaly under conditions of high frequency offset. Examples of these sequences include length 64 ZC sequences indices {1, 21, 63} [2].

As an example of the initial frequency estimation inaccuracy associated with these ZC codes, Figure 5 presents a collection of properties associated with the length 64 index {1} sequence.

As can be seen in Figure 5 all of the down-sampling/decimation phases produce strong correlation peaks that lie either at or at one sample off from the ideal sampling position. However, note in Figure 5, top curve, the very strong dependence that initial frequency estimation accuracy has on decimation/down-sampling phase. By exhaustively considering the individual puncturing of every sequence element it has also been found that in the case of the length 64 ZC sequence indices identified above, that punctured sequence element position has negligible effect on improving the initial/offset frequency estimation inaccuracy observed. Therefore, although these sequences may be very suitable from a time synchronization point of view they seem to be very unsuitable from an initial frequency synchronization point of view.

ZC sequence length 64 index {63} has very similar properties to that of sequence {1} and these are presented in Figure 6. Note that in both these instances {1, 63}, sequence elements have been re-ordered as explained in section 4.

ZC sequence length 64 index {21} however seems to suffer very strongly from the time-frequency ambiguity problem (see the autocorrelation peak shift of up to 20 samples in the plot in Figure 7), and has multiple significant autocorrelation peaks. Initial frequency estimates based on it are more accurate but nevertheless still has a strong dependence on sampling phase (Figure 7). Note in Figure 8, for this same sequence, that the autocorrelation magnitude was found by combining the magnitudes of four 16 element correlations and this restores the maximum peak position to the wanted position. From the multiple correlation peaks that are present, this sequence does however seem less than ideal for primary synchronization purposes.

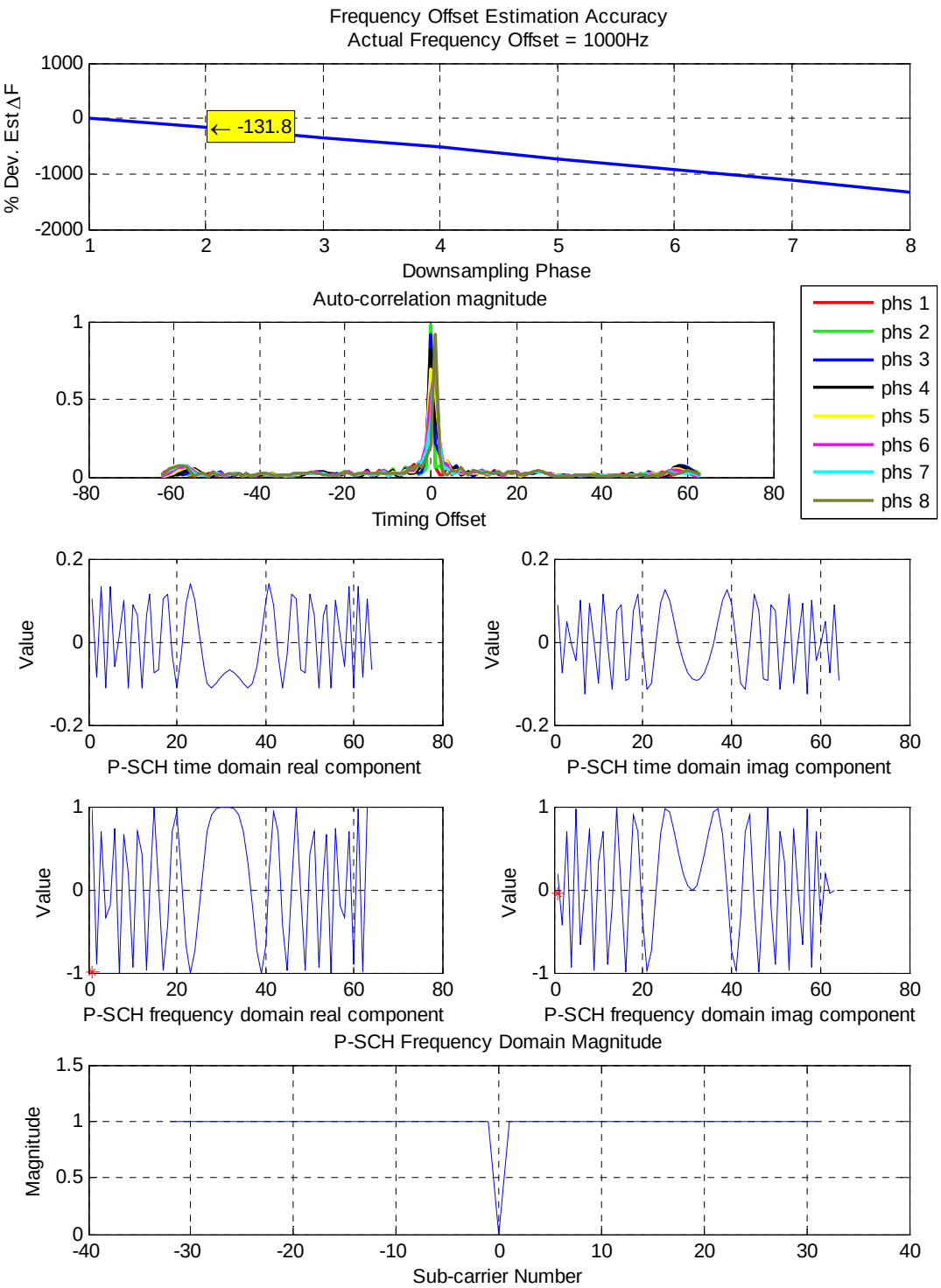


Figure 5: Length 64 ZC sequence index {1} properties. Re-ordering of original ZC sequence elements applied plus puncturing in the position marked with red *.

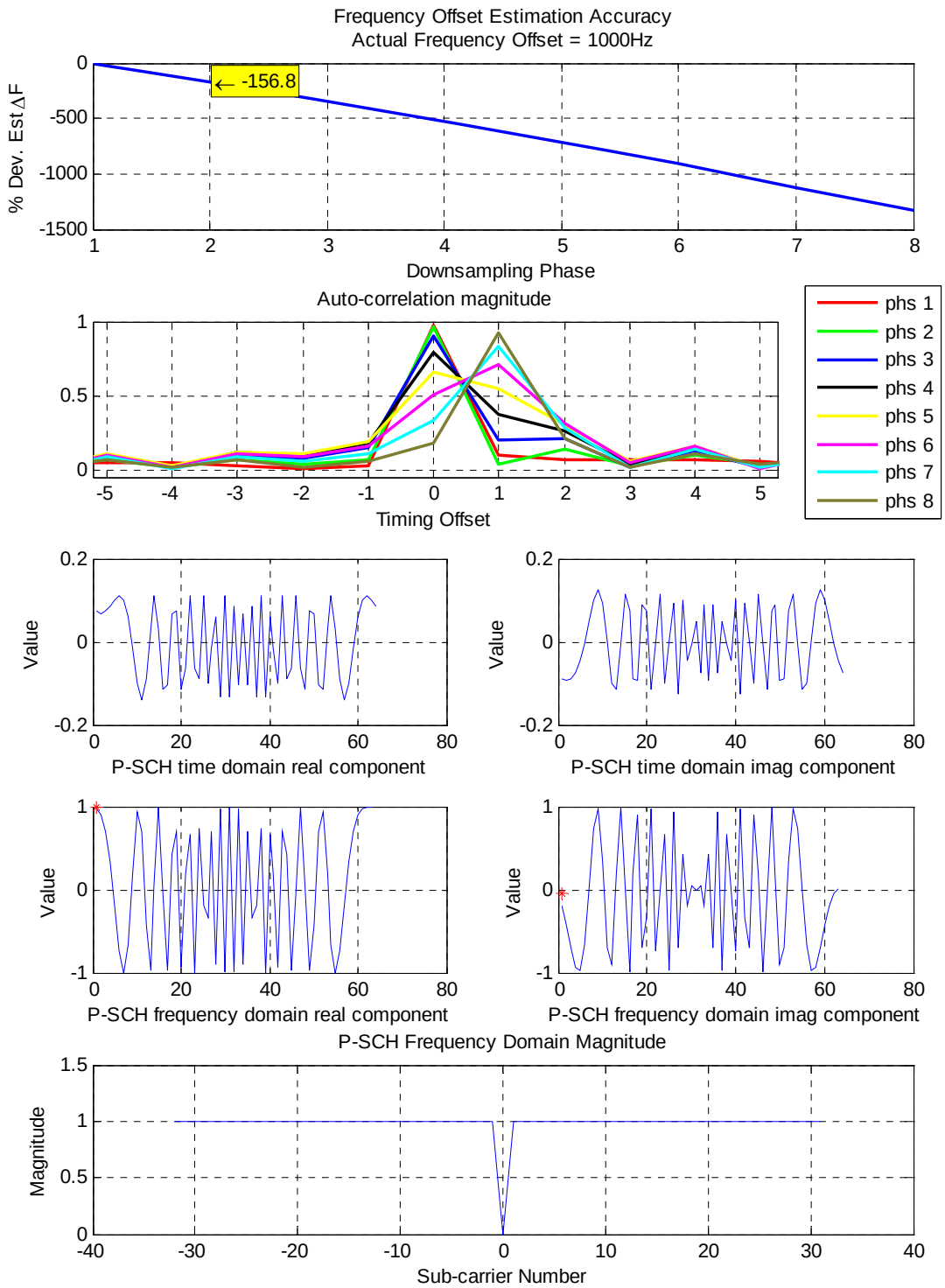


Figure 6: Length 64 ZC index {63} properties. Re-ordering of original ZC sequence elements applied plus puncturing in the position marked with red *.

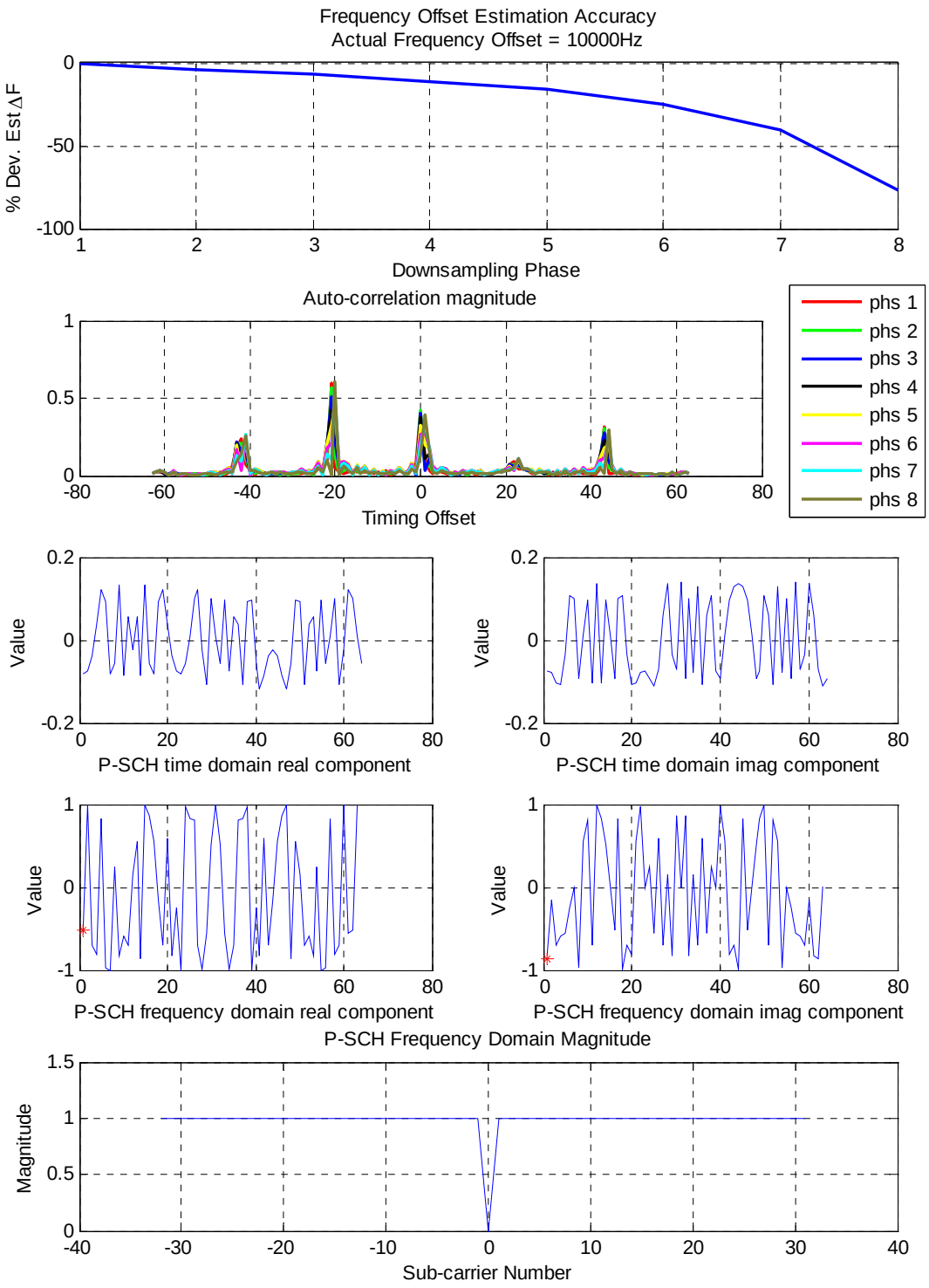


Figure 7: Length 64 ZC sequence index {21} properties. Correlation magnitude found via one length 64 correlation performed coherently. Note the multiple correlation peaks and maximum peak shift as a result of 10kHz frequency offset and coherent full-length matched filtering.

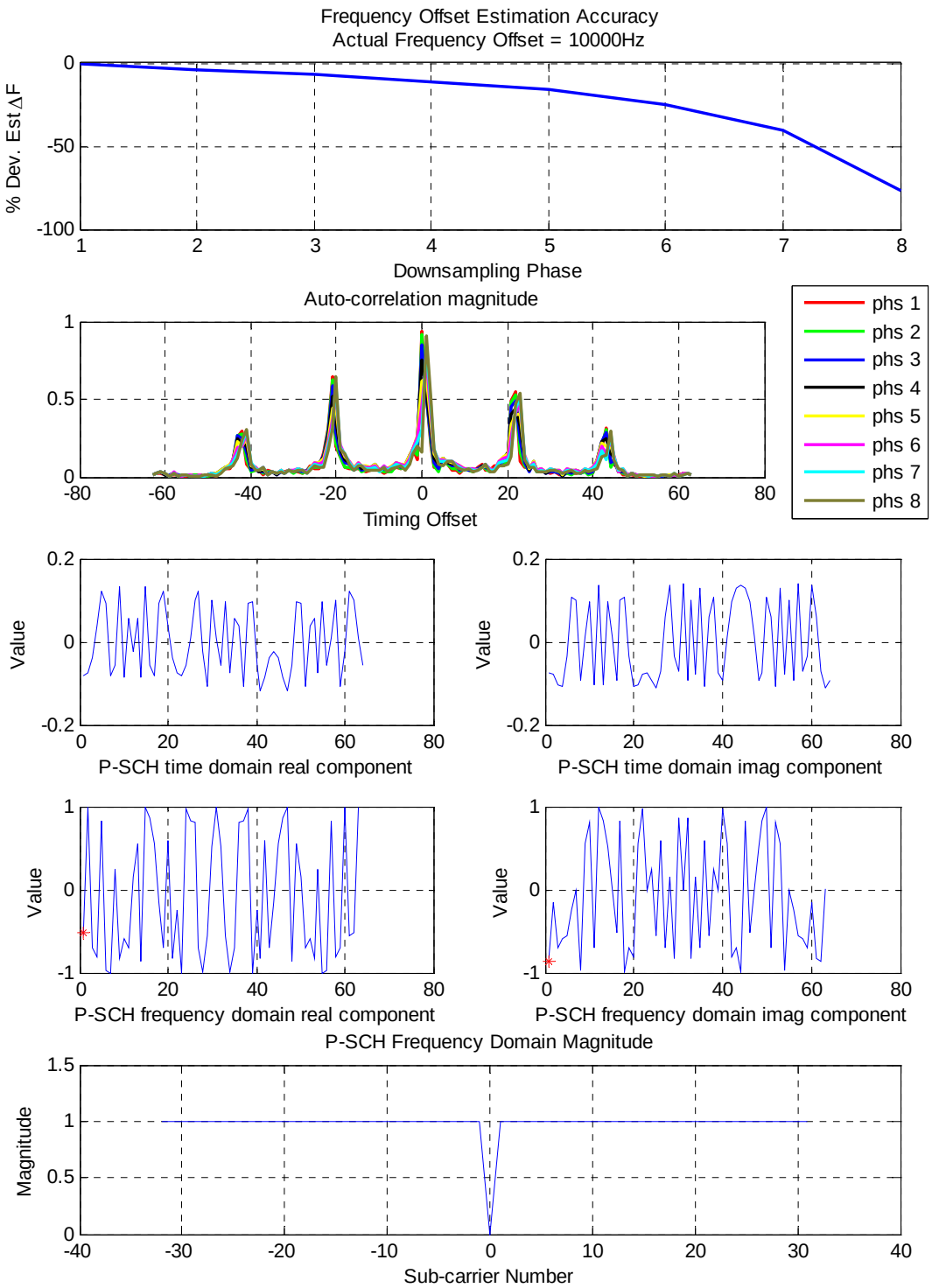


Figure 8: Length 64 ZC sequence index {21} properties. In this case the autocorrelation magnitude is found by combining the magnitudes of four partial correlation magnitudes each of length 16.

7. RESULT: Frequency Domain interleaved ZC Initial Frequency Estimation

Frequency domain interleaved ZC sequences as proposed in [1] have also been investigated with respect to this phenomenon. Once again, in keeping with the assumed working assumption of 64 assigned carriers it was decided to investigate this for an interleaved ZC sequence consisting of two length 32 sequences and the subsequent puncturing of one resultant sequence element from the interleaved sequence and assignment to the centre 64 sub-carriers including the DC sub-carrier. The first sequence is mapped to odd numbered elements and the second sequence is mapped to even numbered elements of the length 64 sequence. Therefore the sequences used here differ slightly from those in [1] however the approach followed is the same as that in [1] except for the repetition of the first sequence element being replaced by puncturing of an element. By varying the puncturing position the most optimal sequence with respect to minimizing initial frequency estimation error can be found.

The results for the interleaved ZC sequence consisting of the two length 32 ZC sequences, indices {19, 21}, are presented in Figure 9. Apart from the puncturing applied, re-ordering of length 64 elements was also applied. Note the favourable frequency offset estimation error performance. However, such performance is dependent on the position at which the original interleaved sequence was punctured. In this case puncturing was performed on interleaved re-ordered sequence element number 45. The puncturing position within the sequence is indicated in Figure 9 on the frequency domain sequence element assignment figure by a red *. The red * represents the element that has been removed from the sequence.

Figure 10 presents the information for the same sequence. However, in this instance the puncturing was performed in a less favourable position than in Figure 9 and the frequency offset estimate deviates by more than 50% in half of the decimation phases.

Therefore, for interleaved, punctured ZC sequences the frequency estimation in-accuracy phenomenon can be overcome by carefully selecting the re-ordering and puncturing position of the sequence element as has been demonstrated in the examples presented above.

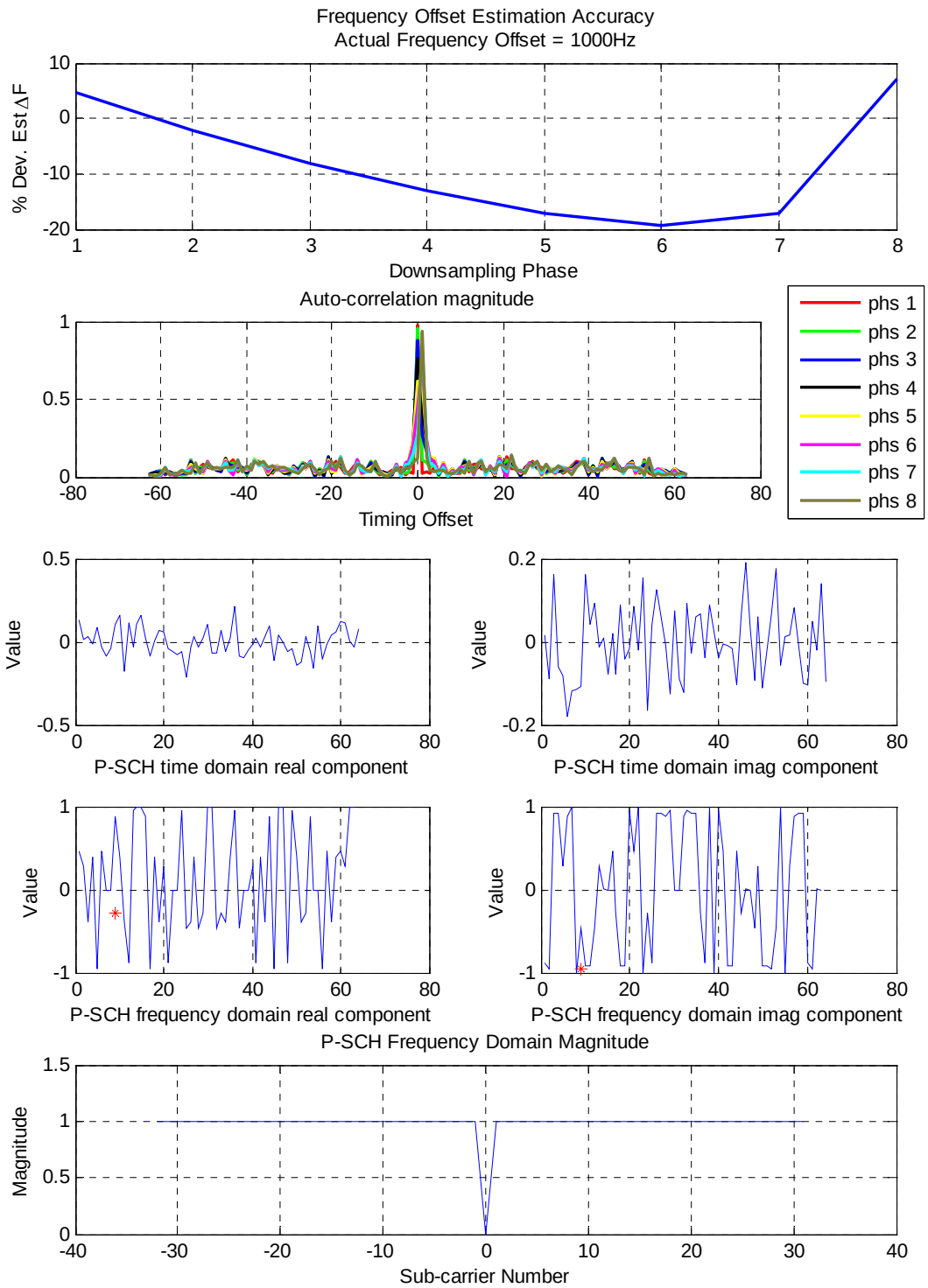


Figure 9: Properties of a length 64 frequency domain interleaved ZC sequence consisting of the two ZC length 32{19,21} sequences. Note the improvement in performance w.r.t. frequency estimation accuracy over all phases. All estimates are within 20% of actual offset. In the case of this sequence the puncturing position within the length 64 interleaved sequence substantially influences the frequency estimation performance. The red * indicates the punctured position within the sequence. Re-ordering of original sequence applied.

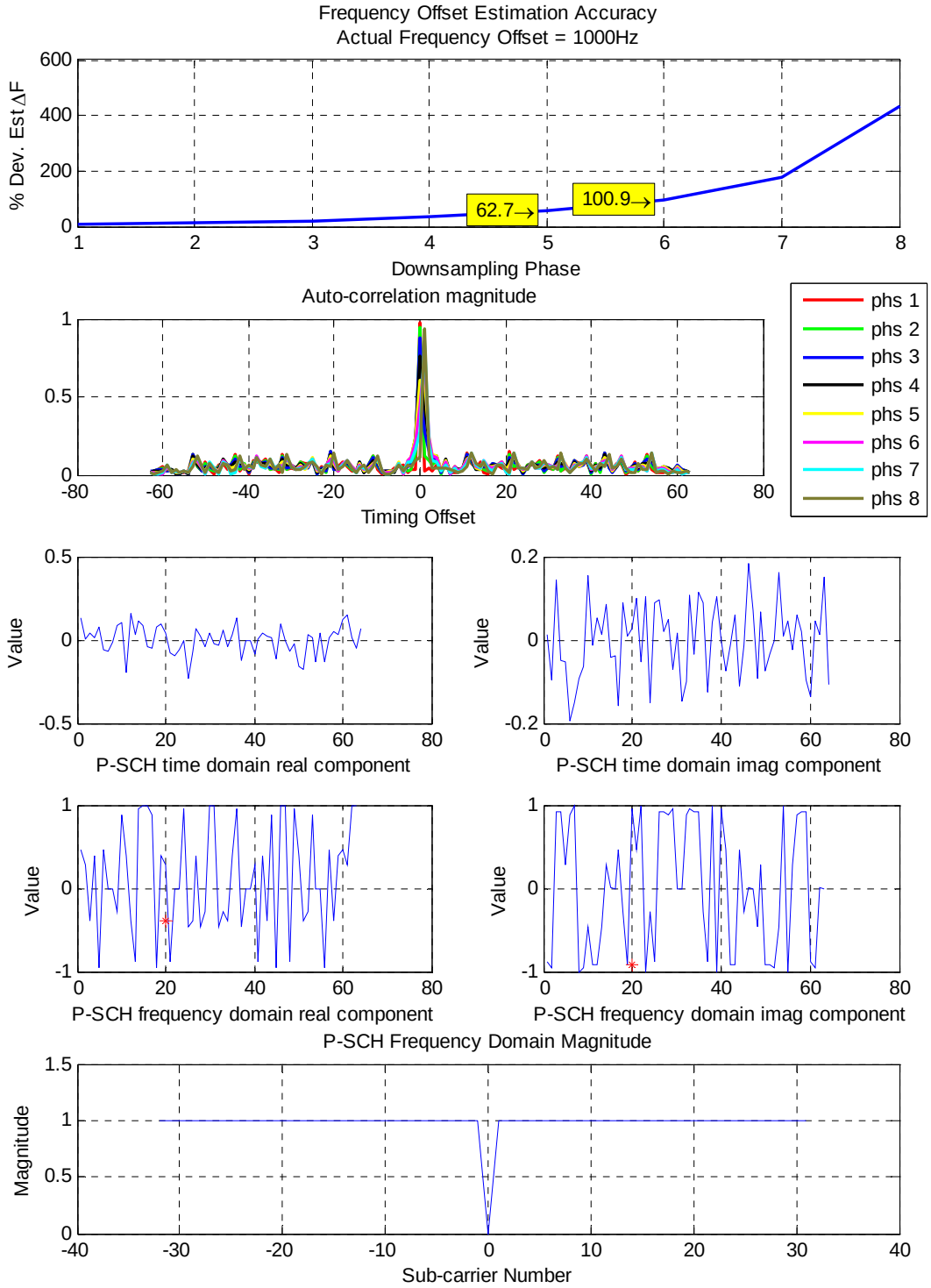


Figure 10: The interleaved ZC length 64 sequence consisting of the two ZC 32{19, 21} sequences with a less favourable puncturing. Note that half of the down-sampling phases lead to grossly erroneous frequency offset estimates deviating by more than 50% from the actual value. Puncturing position and element indicated by means of the red *. Re-ordering of original sequence applied.

8. Constant Amplitude Random Sequences

Considering the nature and properties of the interleaved ZC sequences discussed and presented in the previous section, it is clear that these sequences have a more random nature than the non-interleaved ZC sequences. Hence it would seem that if random sequences were to be used as primary synchronization sequences that the frequency estimation inaccuracy associated with the down-sampling of pure ZC sequences might be mitigated. The use of the P-SCH as a means of supporting coherent demodulation of the S-SCH calls for constant amplitude signal assignment across P-SCH sub-carriers. Therefore it is important, when considering random sequences, to impose on them a constant amplitude (magnitude) constraint.

One possible means of generating constant unit amplitude random sequences is as follows:

1. Choose sequence length, L, to be generated.
2. Generate L uniformly $N(0,1)$ distributed real random variables, $x_i, i = 0,1,\ldots,L-1$, subject to the constraint that $|x_i| \le 1, i = 0,1,\ldots,L-1$.
3. Determine $y_i = \sqrt{1-x_i^2}, i = 0,1,\ldots,L-1$.
4. Generate L uniformly distributed real random variables, $z_i, i = 0,1,\ldots,L-1$, and set $\mathrm{sgn}(y_i) = \mathrm{sgn}(z_i)$.
5. A Constant Amplitude Random(CAR) complex sequence of length L and unit magnitude is now given by $s_i = x_i + jy_i, i = 0,1,\ldots,L-1$.

9. RESULTS: Constant Amplitude Random Sequences

Consider the example of a CAR sequence constructed according to the method outlined in section 8. The actual sequence is presented in the Appendix. As in the case of sequences presented in earlier sections, here too, the position in which the original sequence is punctured is very critical. Nevertheless, as can be seen from Figure 11, a random sequence and puncturing position can be found which leads to very favourable performance(i.e. estimates are within 20% of actual) in terms of frequency offset estimation. In this case, element number 57 of the original sequence has been punctured and it is marked with a red * in Figure 11. Note that in this case element re-ordering was NOT appled.

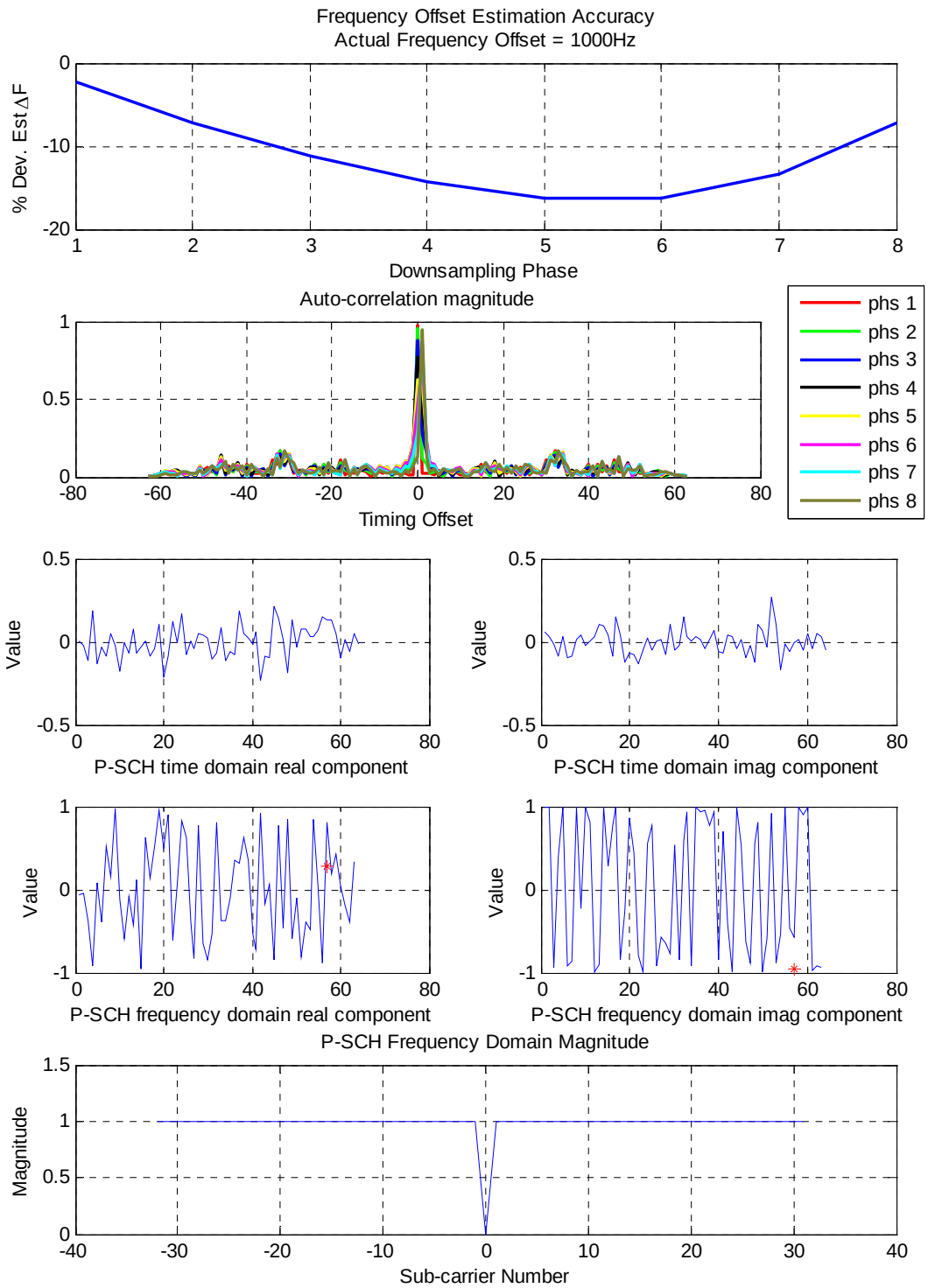


Figure 11: Punctured Constant Amplitude Random Sequence of length 64 with the element at position 57 punctured (red *) from the originally generated CAR sequence. Note that estimated offset frequency falls within 20% for all decimation phases. This particular sequence also has a very strong dependence upon the position in which puncturing of the original sequence is performed. Element re-ordering not applied.

10. Eliminating the dependence on Down-Sampling Phase

One possibility to remove any dependence on down-sampling phase is to perform the correlation at full rate, i.e. not to perform any down-sampling/decimation. Although this makes for a very expensive implementation it is nevertheless interesting for the purposes of this investigation to consider this in order to determine whether full-rate sampling will in any way overcome the phenomenon observed. Once again, ZC, interleaved ZC and CAR sequences are considered.

When performing matched filtering at full rate with an over-sampling factor of , say R, the correlation peak will consist of $2R$ samples with the sampling positions immediately adjacent to the ideal timing position poorly discriminated from the ideal position based on correlation magnitude, the discrimination becoming poorer with an increase in R. The benefit of over-sampled matched filtering though is the fact that timing resolution is increased by a factor of R. Ideally, in this situation, the frequency offset estimates derived should be accurate and independent of timing position over $\pm R$ samples relative to ideal.

For the ZC sequence length 64 index {1} considered earlier in this document, full rate correlation and its associated properties are presented in Figure 12. As can be seen, initial frequency offset estimation accuracy now exhibits a very strong dependence upon timing position relative to ideal. Should full rate correlation be performed on this sequence, the timing positions immediately neighbouring ideal will not be well discriminated from the ideal timing position on the basis of correlation magnitude. Therefore, regardless of whether these sequences are used at base- or full-rate, they always seem to exhibit a strong sensitivity to either timing or down-sampling phase. Therefore ZC sequences of this nature should be avoided based on the requirement of robustness and accuracy of initial frequency estimation.

For the interleaved ZC sequence previously considered, performing correlation at full rate exhibits a slight dependence on sampling time relative to ideal, however, over the poorly discriminated timing positions contending for maximum amplitude i.e. -2 to +2 relative to ideal, the frequency estimation error is accurate to within approximately 10%. This is presented in Figure 13.

For the CAR sequence investigated earlier it was found that it too has less of a sensitivity to timing relative to ideal although not as good as that of the interleaved ZC sequence considered. This is presented in Figure 14.

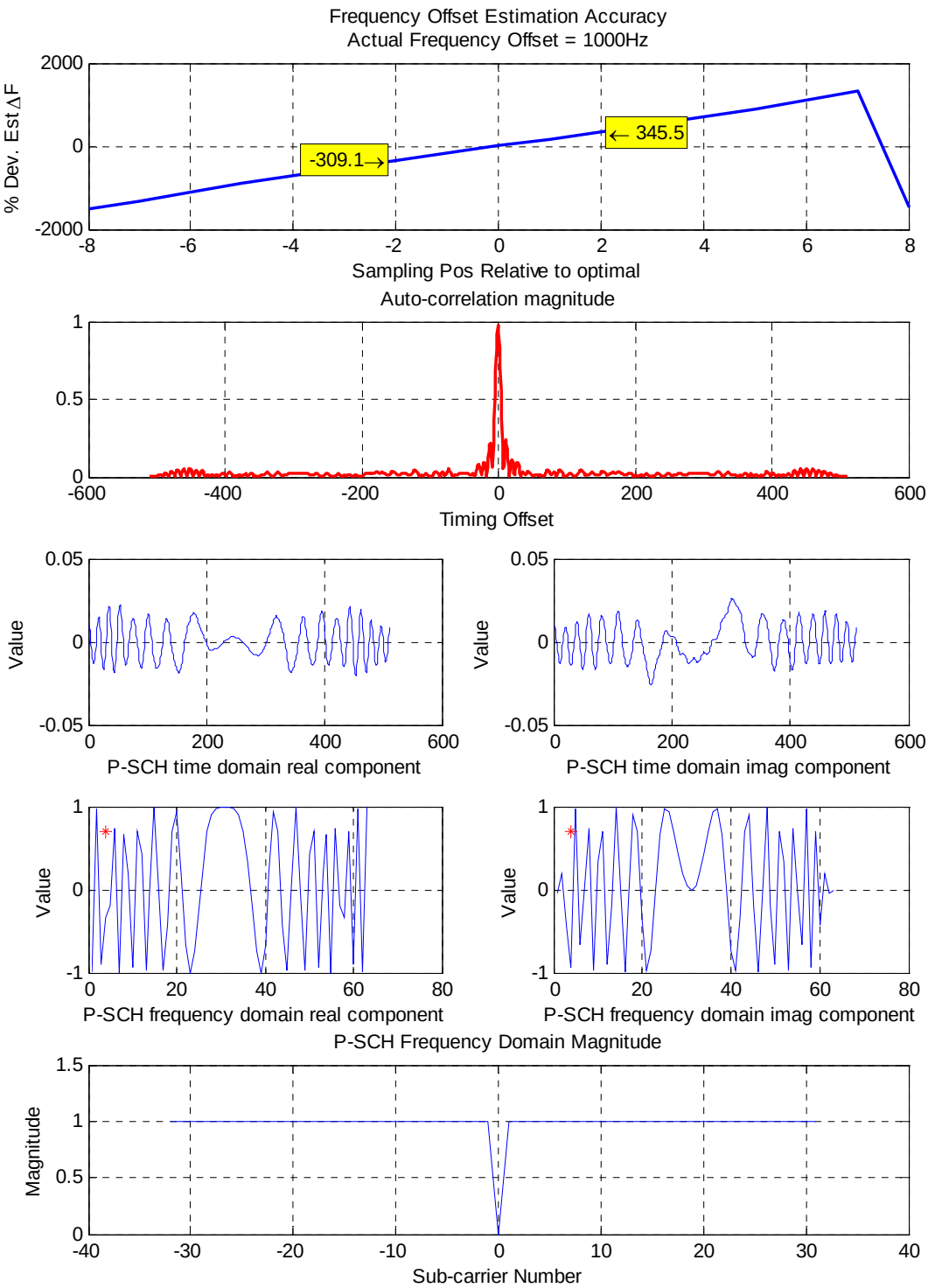


Figure 12: ZC length 64 index{1} properties when full rate correlation is performed. Notice that in this case the frequency offset estimation accuracy problem now appears as an extreme sensitivity to timing position relative to optimal, rather than a down-sampling phase. For these sequences, element puncturing has little effect.

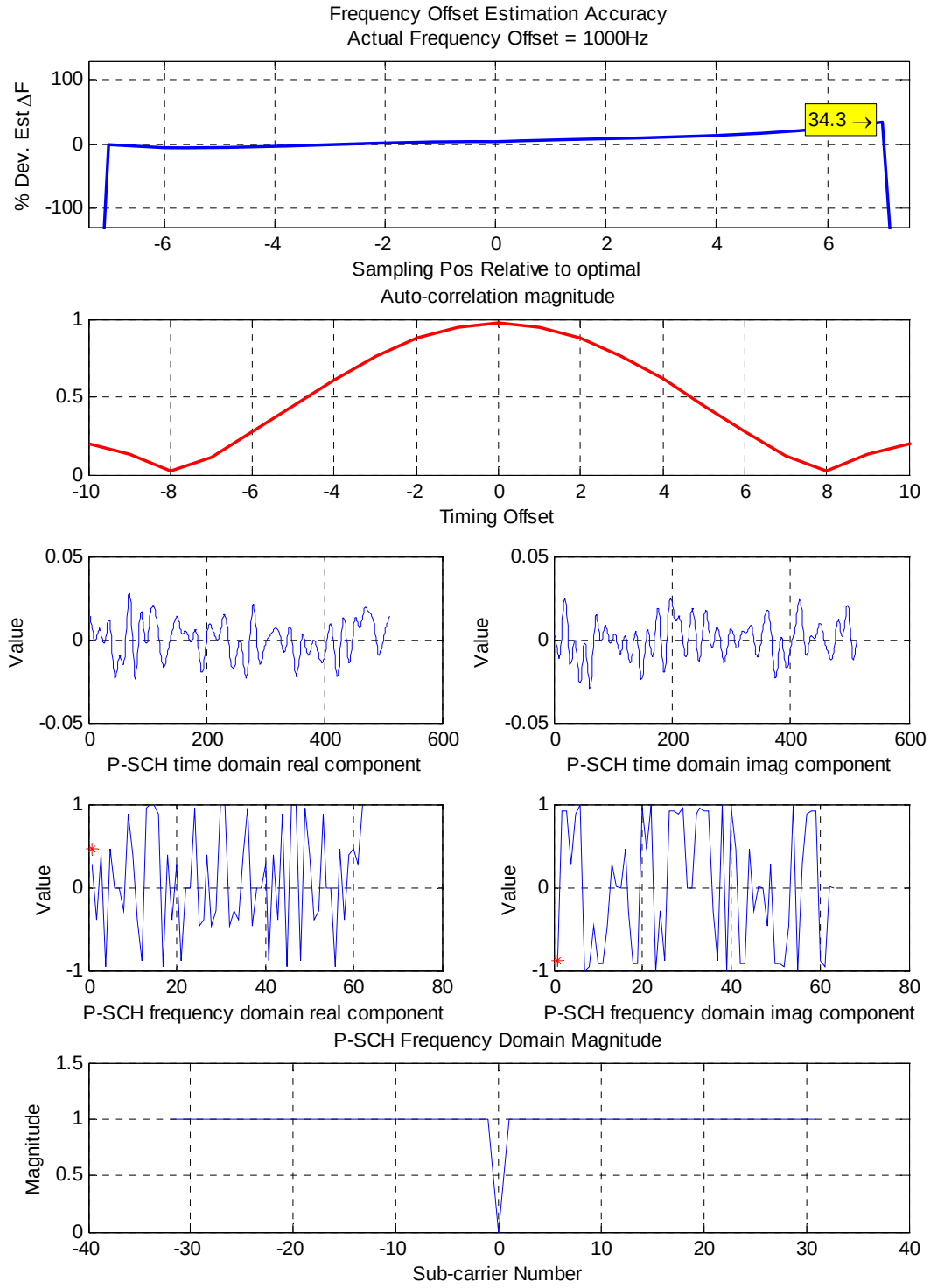


Figure 13: Two interleaved length 32{19,21} ZC sequence properties over-sampled at full rate which in this case is 512 samples per sequence. Note the almost flat frequency offset estimation accuracy curve over -7 to +7 samples relative to ideal. In this case puncturing of the original sequence does not seem to influence the frequency estimation accuracy over the correlation peak span much from that presented here.

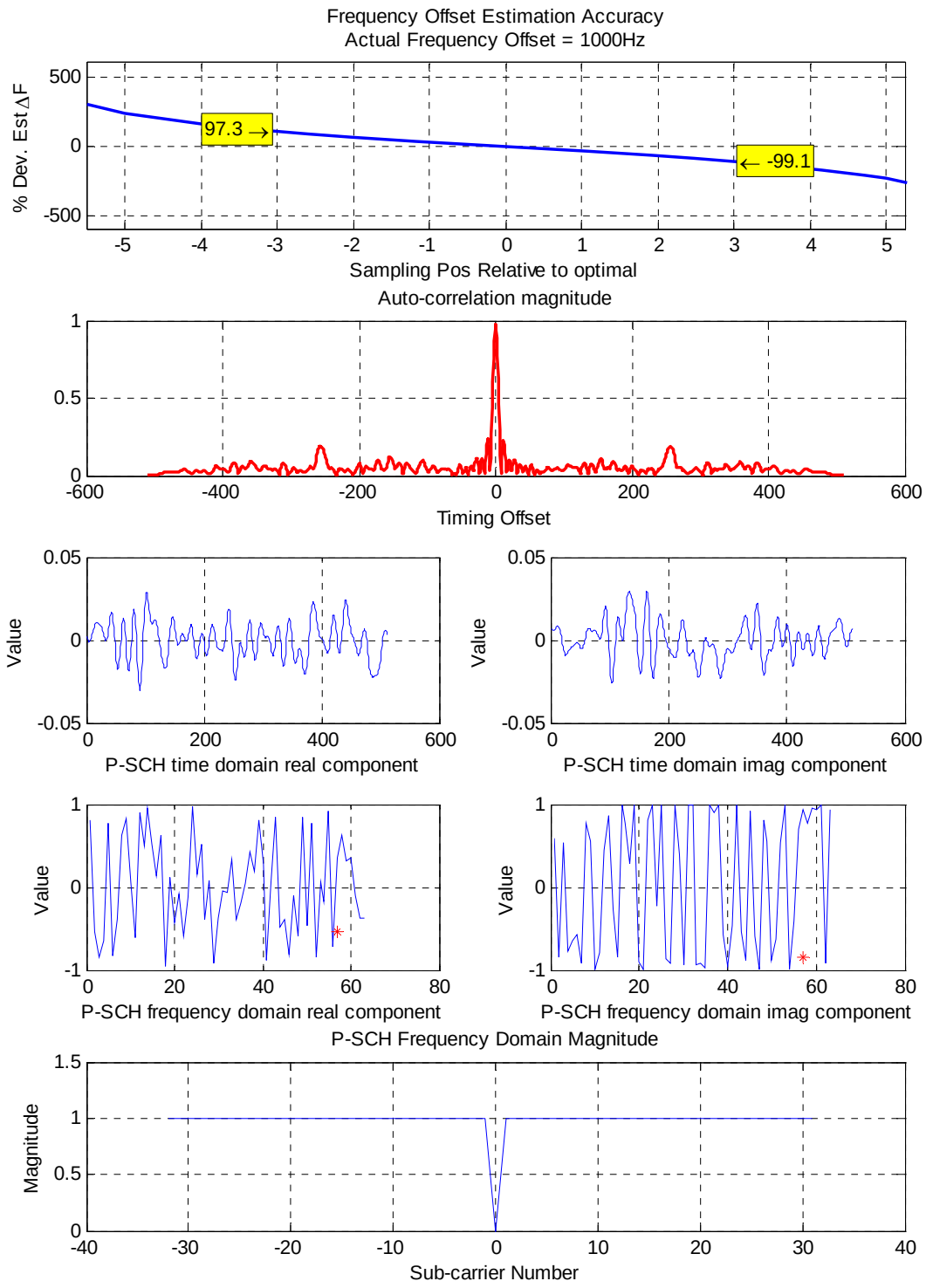


Figure 14: Constant Amplitude Random sequence properties for matched filtering at full rate. Note that for this sequence the frequency offset estimation error shows a stronger dependence on timing relative to optimal than the interleaved ZC sequence considered but much less of a dependence than the ZC length 64 index{1,63} sequences for instance.

11. Conclusions

Although the phenomenon considered in this document touches on receiver architectural issues, it has been shown that depending on the P-SCH sequence selection made, initial synchronization performance is strongly tied to properties of the sequence employed for the P-SCH, irrespective of receiver architecture. Therefore a careful, exhaustive analysis of sequences being considered has to be made in order to specify sequences that will be least restrictive on P-SCH processing and receiver architectures.

Some specific comments:

1. For ZC sequences (length 64 in particular and most likely all lengths in general) the sequences with the most favourable aperiodic autocorrelation properties seem to have the strongest dependency on either down-sampling phase or sampling position relative to optimal when frequency offset estimation is considered. Sequence element puncturing does not seem to influence the behaviour of these sequences

2. Frequency Domain Interleaved ZC sequences seem to be far more favourable in terms of satisfying good autocorrelation properties as well as being able to support accurate frequency offset estimation.

3. Constant Amplitude Random (CAR) sequences have performance similar to that of frequency domain interleaved sequences. These sequences deviate from the current working assumptions in that they are obviously not ZC based, however they do have constant amplitude which is required for channel estimation for coherent S-SCH demodulation.

4. When considering down-sampled vs. full-rate matched filtering it seems clear that a sequence which exhibits poor frequency estimation accuracy for the one approach will typically exhibit poor performance for the other as well. While in the down-sampling case the down-sampling phases will be selected with equal likelihood, the most likely position to be selected in the full-rate sampled case is the ideal position.

5. The results presented in this document are inconclusive as to which particular sequences are best, especially given the number of factors that could potentially affect frequency offset estimation accuracy, i.e. interleaving/non-interleaving, element reordering, element puncturing, etc. Therefore no specific sequences are recommended at this stage as this is still subject to more exhaustive searches incorporating these factors amongst others. Some good sequences have been identified which limit frequency estimation error, these however are not necessarily the best for the type of sequence identified.

12. References

[1] R1-072130 ZC Sequence Based P-SCH Design Using Non-repetitive Structure, Motorola.
[2] R1-072109 Primary Synchronization Codes for LTE Cell Search, NEC Group.
[3] R1-072646 Report of 3GPP TSG RAN WG1 #49, Kobe, Japan 7 – 11 May, 2007.
[4] R1-072107 On ZC-based PSC and Frequency Offset, Marvell Semiconductor.

13. Appendix

13.1 CAR sequence used in section 9

El #	Real	Imag	El #	Real	Imag
1	-0.0634	0.998	33	-0.3731	0.9278
2	-0.0396	0.9992	34	-0.3768	-0.9263
3	-0.3724	-0.9281	35	-0.1018	0.9948
4	-0.9226	0.3856	36	0.3488	0.9372
5	0.0872	0.9962	37	0.3154	0.949
6	-0.3988	-0.917	38	0.6359	0.7718
7	0.5119	-0.859	39	0.3633	0.9317
8	0.1495	0.9888	40	-0.5449	-0.8385
9	0.974	-0.2267	41	-0.722	0.6919
10	-0.1218	0.9926	42	0.9095	-0.4157
11	-0.5951	0.8036	43	-0.1739	-0.9848
12	-0.0753	-0.9972	44	0.0719	0.9974
13	-0.4294	-0.9031	45	-0.8399	0.5428
14	0.1092	0.994	46	0.7741	-0.6331
15	-0.962	0.2732	47	-0.4555	-0.8902
16	0.6332	0.774	48	0.8375	0.5464
17	0.1375	0.9905	49	-0.5881	0.8088
18	0.5195	-0.8545	50	-0.1048	-0.9945
19	0.9625	-0.2711	51	-0.8036	-0.5951
20	0.5025	0.8646	52	-0.3905	0.9206
21	0.8941	0.4478	53	-0.4871	-0.8734
22	-0.6071	-0.7946	54	0.8492	-0.5281
23	0.0592	-0.9982	55	0.0555	0.9985
24	0.8358	0.5491	56	-0.887	-0.4618
25	0.6238	0.7815	57*	0.2755	-0.9613
26	-0.3926	-0.9197	58	0.8138	-0.5812
27	-0.8226	-0.5686	59	0.1848	0.9828
28	0.765	-0.6441	60	0.4182	0.9083
29	-0.6382	-0.7698	61	0.0483	0.9988
30	-0.8449	0.5349	62	-0.1925	-0.9813
31	-0.5389	-0.8423	63	-0.3863	-0.9224
32	0.8109	0.5852	64	0.3391	-0.9408

*Element # 57 is the element punctured from this sequence before assignment to the centre 64 sub-carriers (including DC)