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Retransmission Strategies for MBMS

1 Introduction

In [1] the minimum features required in order to support MBMS service in GERAN are analysed. In Section 2 of [1] it is stated that download-and-play as well as media streaming are the services that will generate the most MBMS traffic. Whereas codecs of the most streaming services can accept residual error rates in the region of 0.1-1%, MBMS services for download-and-play should operate error-free. An advantage of download-and-play services is that these services can be delivered at bit rates different than the content rate. In a common p-t-p GPRS transmission the acknowledged mode bearer would be used to transmit background and file data, as neither retransmission delay nor real-time playout is critical for download-and-play applications such as MMS.

Several proposals [2],[3],[4] have been presented at GERAN #16 which introduce an outer coding scheme based on Reed-Solomon codes on the RLC layer as well as on the BM-SC layer to achieve reasonable throughput at SDU error rates of about 10^{-2} to 10^{-3}. These Forward Error Correction (FEC) schemes are very promising for the specification of an unacknowledged bearer within MBMS. However, for file and data download an acknowledged bearer is required which provides sufficient reliability. This can only be achieved by exploiting feedback and retransmissions.

In contrast to previous considerations proposing the usage of individual p-t-p connections to retransmit lost RLC frames, at GERAN2#16bis in [5] an alternative has been presented. A Common Feedback Channel (CFCH) is introduced which allows informing the BS about lost RLC packets and to enable retransmissions within the p-t-M connection. Preliminary simulation results for this proposal were shown in [10].

In this document we compare several retransmission strategies which are under consideration to define an MBMS acknowledged mode bearer. First we compare p-t-p retransmissions with p-t-M retransmissions as proposed in [5] and [6] and show that significant gains are obtained in terms of throughput by introducing and exploiting the CFCH. Moreover, we propose FEC on RLC layer with Reed-Solomon codes in combination with feedback exploitation on the CFCH in order to design the MBMS acknowledged mode bearer. We show that this approach outperforms the p-t-p retransmission scenario, as well as the plain p-t-M retransmission scenario, and, that the performance is only slightly depending on the number of served user terminals in the contrast to the other schemes, which degenerate significantly with increasing number of users.

2 Retransmission Strategies

In this section we review already proposed Automatic Repeat reQuest (ARQ) strategies and propose the combination of outer coding in the RLC in combination with feedback exploitation in order to define the MBMS acknowledged mode bearer. For all presented retransmission schemes we consider the BS to broadcast or transmit in persistent mode, i.e. an RLC frame is retransmitted or redundancy is broadcasted until the RLC frame under consideration is received correctly by all receivers. In practice the maximum number of retransmission attempts would be limited such that

only those users with sufficient C/I, e.g. C/I>7dB error free service can be guaranteed. We derive the average **error free** throughput of the MBMS service in kbit/s for each ARQ scheme.

p-t-p Retransmissions

In [1] it was already proposed that for the support of download services with negligible residual error rate, two steps to transmit one file to several users in a cell are necessary. Firstly, a p-t-M connection using a MDTCH to all terminals is established and the data is transmitted in best-effort manner. Secondly, after closing the p-t-M session all terminals which have detected lost RLC frames establish p-t-p connections in acknowledged mode and request all lost packets individually. All lost packets are retransmitted in separate p-t-p connections to the terminals. Obviously, this is a straightforward approach, as p-t-p connections are already supported within GPRS. However, they generally provide low efficiency for distributing identical content to several users, due to the fact that identical packets to be transmitted to M terminals will require M times more radio resources. Obviously, all sequence numbers of all lost packets have to be stored at the receivers until the end of the p-t-M broadcast session, what will require appropriate storage at the terminals, as the p-t-p retransmission session is likely to be performed after the p-t-M session.

We consider the throughput assuming M user terminals in the serving area, where each user m experiences an individual RLC frame error probabilities corresponding to a certain C/I and a certain coding scheme. In Annex A the detailed derivation of the throughput for this scenario is provided. However, in the following we restrict ourselves to the special case, where all terminals experience same C/I as the worst case approximation. Certainly, if some mobiles are experiencing higher C/I the throughput will be increased and can be obtained by evaluating the equations in Annex A. In this special case, the average throughput of the p-t-p retransmission scenario assuming equal RLC frame error probability for all user terminals is given by

$$\eta(M,p) = \frac{0.4 \cdot L(\mathrm{CS})}{1 + M\frac{p}{1-p}} \text{ in kbit/s,}$$

with M the number of users in the cell, p denoting the RLC frame error probability and L(CS) the payload size of the applied RLC coding scheme in byte. Please note that for all further presented retransmission strategies in the next subsections the same notations and considerations are valid and that $\eta(M,p)$ specifies the error-free throughput. Note that in this case the throughput decreases with the number users with order $O(M(p/1\text{-}p))$, i.e. linearly with the number of user M.

p-t-M Retransmission

As outlined in the previous subsection p-t-p retransmissions do not seem to be very efficient, due to packets with identical sequence numbers to be retransmitted wasting multiple times the radio resources. Especially for scenarios with many users, e.g. in a football stadium, this transmission methods gets very wasteful. The basic idea of p-t-M retransmissions is that packets to be retransmitted are also broadcasted over a common physical channel in the downlink. This requires an appropriate uplink feedback channel to signal lost frames from the terminal to the BS. A promising approach is presented in [5] and [6] with the introduction of the Common Feedback Channel (CFCH), where all terminals utilize a common timeslot and a common frequency to signal unsuccessful packet receptions. The feasibility and the detailed syntax of the CFCH has to be

evaluated in the future, but for now we consider its semantics as accessible and show the possible gains by its usage. The semantics allow sending a NAK for each transmitted message for each user to the transmitter, ACKs are not sent at all.

The major benefit of p-t-M retransmission systems results from the fact that a specific packet which was requested by several terminals to be retransmitted will require only one packet transmission for its retransmission, rather than *M* channel accesses assuming *M* terminals requesting this packet in a p-t-p retransmission environment. This obviously results in increased system efficiency. In general, the number of channel uses, or required packet transmissions, for p-t-M is less or equal to the p-t-p retransmission scenario. Equality is given if disjoint packets are requested by the terminals, which is not expected to happen very often, especially in case more users. Furthermore, one major advantage of the p-t-M retransmission scenario in contrast to the p-t-p retransmission scenario is that the retransmissions can be performed immediately after an unsuccessful packet transmission, as the transmission bearer for transmission and retransmission is identical. This reduces the memory requirement at the receivers, as sequence numbers of packets do not have to be stored until the end of the file transmission. According to Annex A and considering the setup and notation already introduced in the previous subsection the average throughput of the p-t-M retransmission scenario can be derived as.

$$\eta(M,p) = \frac{0.4 \cdot L(\mathrm{CS})}{\sum_{t=1}^{\infty} t \cdot \left(\left[1 - p^{t}\right]^{M} - \left[1 - p^{t-1}\right]^{M} \right)} \text{ kbit/s.}$$

Please note that $\eta(M,p)$ specifies the error-free throughput.

p-t-M Retransmissions exploiting incremental redundancy (p-t-M-IR)

In [7] it was shown that the performance of p-t-M retransmission systems can be significantly improved by exploiting outer coding with incremental redundancy[1]. Instead of retransmitting the lost packet itself, a redundancy packet is broadcasted. A similar approach is used for hybrid ARQ protocols in point-to-point connections. In the p-t-M scenario the transmission of incremental redundancy, in contrast to simple packet repetition, reduces the number of required channel uses further.

The proposed system in [7] exploits outer coding with punctured systematic Reed-Solomon codes to recover from packet losses in a p-t-M environment. This approach seems to be very promising for the definition of an MBMS acknowledged mode bearer. Furthermore, it is compatible with the proposed flexible outer Reed-Solomon coding on RLC layer [8]. To be more specific, it is a combination and enhancement of both systems, the RLC outer coding with Reed-Solomon codes [8] and to the p-t-M retransmission scenario [5], [6].

Any (n,k) Reed-Solomon (RS) code as proposed in [8] is applied to obtain (n-k) redundancy packets from k packets (RLC frames) to be broadcasted. For a detailed explanation and description of the RS encoding policy and interleaving, refer to [8] and [9]. The encoding is performed such

[1] Please note, incremental redundancy in this context does not address using EGPRS coding schemes and performing incremental redundancy on physical layer, but rather the exploitation of an outer code on RLC layer over multiple RLC frames which provides the possibility of forming redundancy packets, which are transmitted incrementally when required.

that for each receiving terminal it is sufficient to receive any k packets (RLC frames) out of a set of n packets, to recover all k information packets (RLC frames). Therefore, in the p-t-M-IR scenario, first, k RLC packets are broadcasted from the BS to the receiving terminals. User terminals which could not receive all k packets correctly indicate this to the BS using the NACK on the CFCH. Terminals which have received all k RLC frames correctly do not indicate anything, according to [5] and [6]. As a reaction on a feedback message received on the CFCH, the BS broadcasts one[2] redundancy packets (RLC frames). This is continued until all user terminals are able to recover all k information packets. According to Annex A and considering the setup and notation already introduced in the first subsection of this document the average error free throughput of the p-t-M-IR retransmission scenario for the case where one redundancy packet is broadcasted after each negative acknowledgement can be determined as

$$\eta(M,p) = \frac{0.4 \cdot L(\mathrm{CS}) \cdot k}{\sum_{t=k}^{\infty} t \cdot \left\{ \left[1 - \sum_{i=0}^{k-1} \binom{t}{i} (1-p)^i \, p^{t-i} \right]^M - \left[1 - \sum_{i=0}^{k-1} \binom{t-1}{i} (1-p)^i \, p^{t-i} \right]^M \right\}} \text{ in kbit/s.}$$

As the CFCH is used the first time after k RLC frames were broadcasted, the feedback ratio for this scheme is significantly lower than in the case without incremental redundancy. This automatically yields in reduced interference, resulting from multiple terminals using the CFCH at the same time, which has to be evaluated in further detail.

3 Performance Evaluation

In this section the achievable error free throughput for single radio time-slot of all presented retransmission scenarios is compared. For this, we have used RLC frame loss rates over a TU03 channel with frequency hopping at different C/I. This is summarized in Table 1 along with the used payload size L for each GPRS coding scheme (CS).

Table 1 Payload size and RLC/MAC block loss rates at different C/I, different coding schemes with frequency hopping.

	Payload size *L*	7.5 dB FH	10 dB FH	12.5 dB FH
CS 1	20 bytes	0.119	0.030	0.008
CS 2	30 bytes	0.357	0.150	0.051
CS 3	36 bytes	0.502	0.271	0.108
CS 4	50 bytes	0.912	0.777	0.598

In Fig. 1 the achievable throughput for p-t-p retransmissions and plain p-t-M retransmissions, exploiting the CFCH, over the number of users M is shown for C/I=7.5dB, coding scheme CS1 and ideal frequency hopping. Both schemes degenerate in terms of average throughput with increasing number of users. However, the degeneration of the p-t-M scheme is significantly lower, which results in 3 times higher throughput compared to p-t-p case for M=50 users. Moreover, the

[2] Note that in general a coarser granularity than 1 for number of packets in a retransmission attempt could be introduced. However, in this initial work we only consider the throughput optimal case with a feedback after the transmission of each redundancy packet.

extremely low throughput for the p-t-p retransmission scheme of 1 kbit/s does not seem to be sufficient to define an MBMS acknowledged bearer, as with increasing number of users the degeneration is enormous.

In Fig. 2 the average throughputs of the p-t-M-IR retransmission scheme for Reed Solomon code parameters k=12 and k=128 are compared to the p-t-p as well as to the p-t-M retransmission scenario, for C/I=7.5dB, coding scheme CS1 and ideal frequency hopping. The average throughput for p-t-M-IR is approximately doubled compared to the plain p-t-M scheme and even six times higher than the p-t-p throughput when assuming M=50 users in the cell. Furthermore, the throughput degeneration for p-t-M-IR with increasing number of user terminals is significantly lower compared to p-t-p and plain p-t-M. For CS1, k=128 and C/I=10dB (Fig. 5) the throughput is virtually constant with the number of terminals. This results from the low RLC frame error probability in combination with the excellent performance of the proposed incremental redundancy scheme. Finally, Fig. 3 - Fig. 5 compare all presented retransmission schemes for all coding schemes CS1 –CS4 and for different C/I values. The advantages of flexible RS codes as proposed in [8] are also obvious from these results.

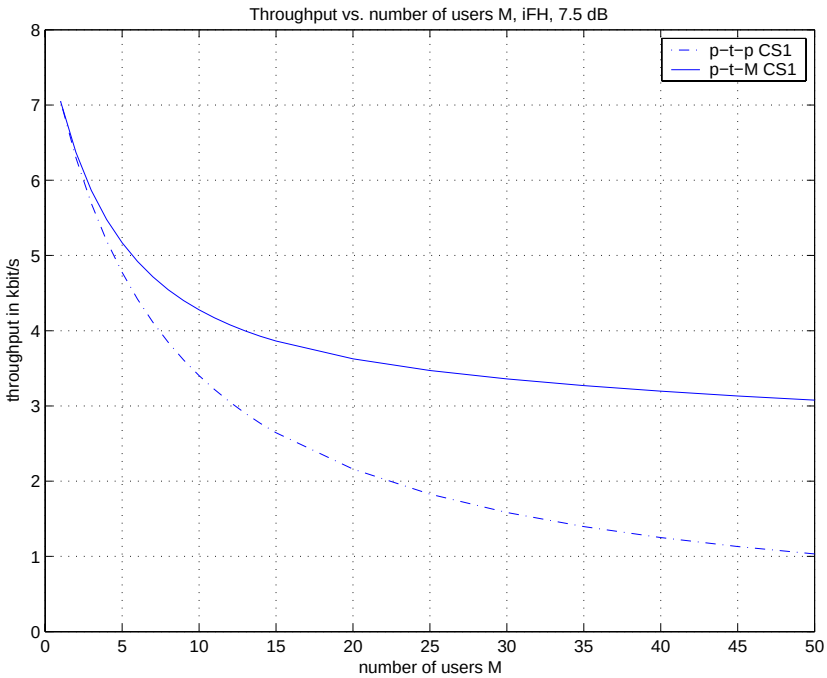

Fig. 1.: Throughput of p-t-p retransmissions and p-t-M retransmissions over number of users M for C/I=7.5 dB and CS1.

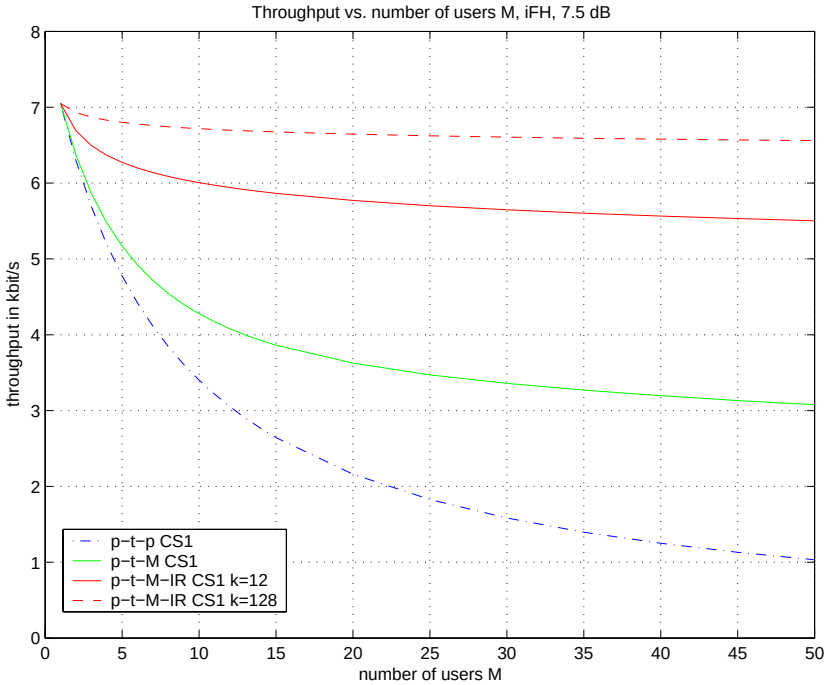

Fig. 2 Throughput of p-t-p, p-t-M, p-t-M-IR k=12 and p-t-M-IR k=128 for C/I=7.5 dB and CS1.

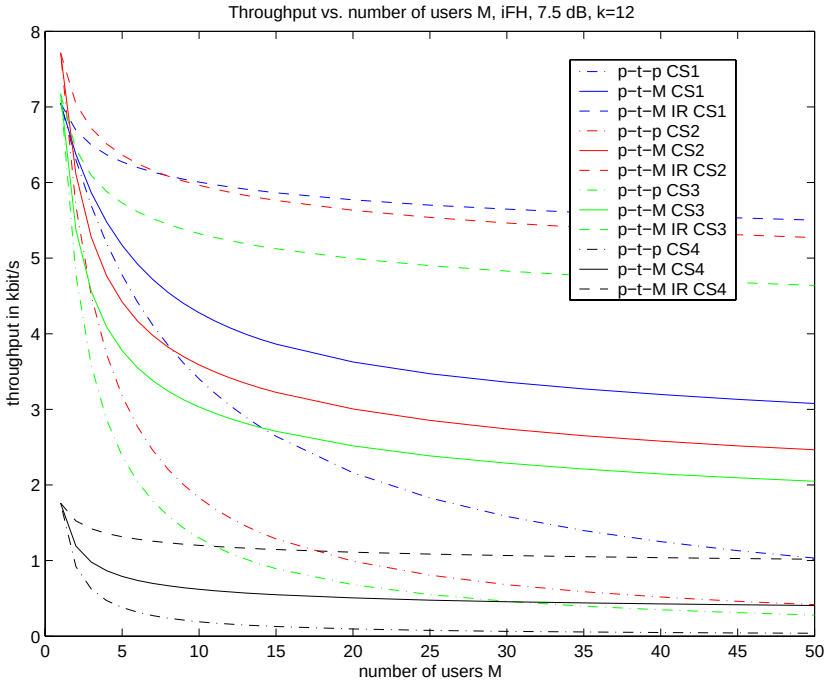

Fig. 3 Throughput of p-t-p, p-t-M and p-t-M-IR k=12 for C/I=7.5 dB and CS1-CS4.

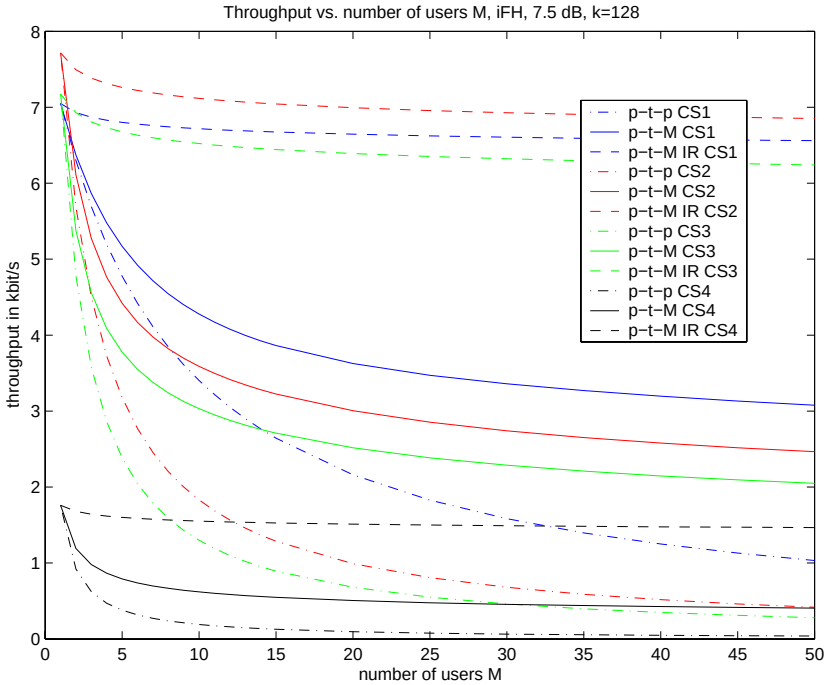

Fig. 4 Throughput of p-t-p, p-t-M and p-t-M-IR k=12 8 for C/I=7.5 dB and CS1-CS4.

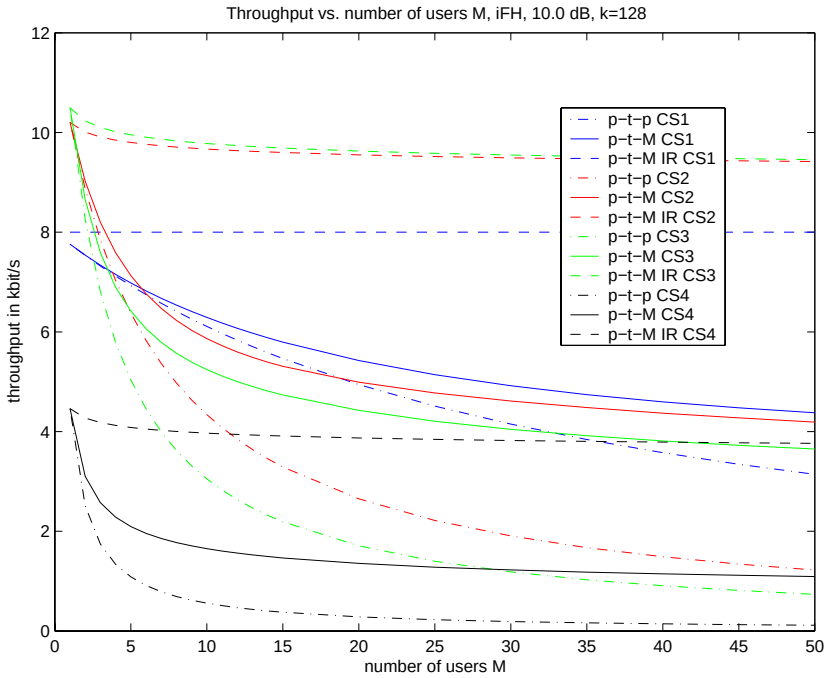

Fig. 5 Throughput of p-t-p, p-t-M and p-t-M-IR k=128 for C/I=10 dB and CS1-CS4.

4 Conclusions

In this document we compared several retransmission strategies which are under consideration to define an MBMS acknowledged mode bearer. First we compared p-t-p retransmissions with p-t-M

retransmissions as proposed in [5] and [6] and showed that significant gains are obtained in terms of throughput by introducing and exploiting the CFCH. It was shown that p-t-p retransmissions cannot provide sufficient efficiency to define an acknowledged mode bearer within MBMS for an increased number of users to be served in a cell. Therefore, the syntax as well as the semantics of the necessary CFCH should be evaluated in further detail.

Moreover, we proposed FEC on RLC layer with Reed-Solomon codes [8] in combination with feedback exploitation on the CFCH [6] in order to design an efficient MBMS acknowledged mode bearer. It was shown that this approach outperforms the p-t-p retransmission scenario, as well as the plain p-t-M retransmission scenario significantly, and, that the performance is only slightly depending on the number of served user terminals in the contrast to the other schemes, which degenerate significantly with increasing number of users. Compared with the plain p-t-M retransmission scheme the number of feedbacks messages from terminals to the BS could be reduces significantly, which yields in reduced interference. This will be studied in more detail in the future.

Introducing FEC with Reed-Solomon codes on the RLC layer can be exploited for both, an unacknowledged MBMS bearer, where no feedback is available and for an acknowledged mode bearer, in the case of feedback and CFCH utilization. Furthermore, the introduction of the CFCH can be utilized to enable plain p-t-M retransmissions or p-t-M retransmissions with incremental redundancy. Therefore, it seems to be beneficial to introduce Reed-Solomon coding on RLC for the definition of an unacknowledged bearer and to use the same RLC FEC coding to define the acknowledged mode bearer.

5 References

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[4] GP-032101, “Outer coding on RLC layer for MBMS over GERAN: Extension to multislot mode”, Siemens, GERAN #16, New York, USA, 25-29 August 2003.

[5] G2-030470, “p-t-M with feedback for MBMS delivery: overview”, Siemens, GERAN2 #16bis, Pova de Varzim, Portugal, 6-10 October 2003.

[6] GP-032604, “p-t-M with feedback for MBMS delivery: overview”, Siemens, GERAN #17, Budapest, Hungary, 17-21 November 2003.

[7] J.J. Metzner, “An improved broadcast retransmission protocol”, IEEE Trans. Commun. Vol. COM-32, pp. 679-683, Jun. 1984.

[8] GP-032614, “Outer coding in the RLC for MBMS”, Siemens, GERAN #17, Budapest, Hungary, 17-21 November 2003.

[9] GP-032613, "Complexity estimate for Outer Coding in the RLC layer: Encoding and Decoding with Reed-Solomon codes", Siemens, GERAN#17, Budapest, Hungary, 17-21 November 2003.

[10] G2-030471, "p-t-M with feedbacks for MBMS delivery: preliminary simulation results", Siemens, GERAN2 #16bis, Pova de Varzim, Portugal, 6-10 October 2003.

6 Annex A

In this section the relative throughput of the proposed ARQ schemes is derived. For reasons of simplicity, in the following an RLC frame is denoted by packet. For all derivations we assume that a received packet at receiver m is erroneous with probability p_m and received correctly with probability $1 - p_m$. Furthermore, it is assumed that packet losses occur statistically independent and that packet losses occur independently among receivers.

P-t-P Retransmissions

In order to calculate the throughput of the p-t-p retransmission scheme the transmission of a single packet from the BS to M user terminals is considered. Let T_m be a random variable denoting the number of required retransmission attempts (channel uses) to transmit one packet correctly to **a single** user m, with $T_m \in \{0,1,2,...\}$ and $m \in \{1,2,...,M\}$. We consider that the initial transmission attempt for this packet is within the multicast session and all retransmissions take place in the separate p-t-p session for this specific user terminal. Hence, the probability that exactly t retransmissions are required is

$$\Pr\{T_m = t\} = p_m \cdot p_m^{t-1}(1 - p_m) = p_m^t(1 - p_m),$$

with $t \in \{0,1,2,...\}$.

Consequently, and by applying

$$\sum_{t=0}^{\infty} t \cdot x^t = \frac{x}{(1-x)^2},$$

the expected value $E\{T_m\}$ for the number of required retransmissions for receiver m results in

$$E\{T_m\} = \sum_{t=0}^{\infty} t \Pr\{T_m = t\} = \sum_{t=0}^{\infty} t p_m^t (1 - p_m) = (1 - p_m)\sum_{t=0}^{\infty} t p_m^t = \frac{p_m}{1 - p_m},$$

which allows to give the relative throughput $\eta(\mathbf{p})$ of the p-t-p retransmission scenario as

$$\eta(\mathbf{p}) = \frac{1}{1 + \sum_m E\{T_m\}} = \frac{1}{1 + \sum_m \frac{p_m}{1 - p_m}},$$

with $\mathbf{p} = (p_1 \quad p_2 \quad ... \quad p_M)$.

P-t-M Retransmissions

We consider M terminals in the cell. Each time a broadcasted packet could not be received by a user terminal the corresponding terminal sends a negative acknowledgment, denoted by *nak*, to the BS. In the case of a correctly received packet no message is sent by the terminals. The BS receives the superposition of all *naks,* sent by all terminals which could not receive the packet correctly. We

denote the superposition of all *naks,* which can also be interpreted as an OR connection of all send *naks*, as *NAK*. In other words, if the BS receives a *NAK*, at least one user terminal sent a *nak*.

Let t denote the number of packet replicas broadcasted to the receivers, which implies t-1 retransmissions of a certain packet. Let $p_{\mathrm{n},m}(t)$ denote the probability that a single receiver sends a *nak* after it received t replicas of the same packet, i.e. the probability of unsuccessful reception of the current packet after t transmission attempts. Hence, $p_{\mathrm{n},m}(t)$ is given by

$$p_{\mathrm{n},m}(t) = p_m^t .$$

Let $p_{\mathrm{N}}(t)$ denote the probability of a *NAK* reception at the BS after t broadcast attempts that equals to the probability that at least one user terminal sends a *nak* after t transmissions. Thus, $p_{\mathrm{N}}(t)$ can be written as

$$p_{\mathrm{N}}(t) = 1 - \prod_m [1 - p_{\mathrm{n},m}(t)] = 1 - \prod_m [1 - p_m^t] .$$

Let T be a random variable denoting the number of required overall broadcast attempts (channel uses), including the first transmission and all retransmissions, to broadcast one packet correctly to **all** users. Let $\Pr\{T > t\}$ denote the probability that more than t transmissions are required to deliver the packet to all receivers correctly, which is given by the probability that after t transmissions a *NAK* is received and, thus there are still user terminals requiring retransmissions. Hence, $\Pr\{T > t\}$ is given by

$$\Pr(T > t) = p_{\mathrm{N}}(t) = 1 - \prod_m [1 - p_m^t].$$

Consequently, the probability $\Pr(T = t)$ that exactly T transmissions are required to transmit the packet correctly to all M users can be written as

$$\begin{aligned}\Pr(T = t) &= \Pr(T > t-1) - \Pr(T > t) = \\ &= p_{\mathrm{N}}(t-1) - p_{\mathrm{N}}(t) = \\ &= 1 - \prod_m [1 - p_m^{t-1}] - \left[1 - \prod_m [1 - p_m^t]\right] = \\ &= \prod_m [1 - p_m^t] - \prod_m [1 - p_m^{t-1}].\end{aligned}$$

Following from this the expected value $E\{T\}$ for the number of required broadcast transmissions can be written as

$$E\{T\} = \sum_{t=1}^{\infty} t \cdot \Pr\{T = t\} = \sum_{t=1}^{\infty} t \cdot \left(\prod_m [1 - p_m^t] - \prod_m [1 - p_m^{t-1}]\right),$$

which allows us to give the relative throughput $\eta(\mathbf{p})$ of the p-t-m retransmission scenario as

$$\eta(\mathbf{p}) = \frac{1}{E\{T\}} = \frac{1}{\sum_{t=1}^{\infty} t \cdot \left(\prod_m [1 - p_m^t] - \prod_m [1 - p_m^{t-1}]\right)} .$$

P-t-M Incremental Redundancy with Reed-Solomon Codes

Suppose a systematic (k,n) Reed-Solomon code is used to create n-k redundancy packets and suppose that each packet is in correspondence to one Reed-Solomon code symbol. We assume n to be sufficiently large in comparison to k. First, we consider k packets to be broadcasted to the user

terminals. Second, after k packets were broadcasted, the BS sends redundancy packets if required. Let t denote the number of packets broadcasted from one code word. User terminals which could not receive all k information packets correctly after t packets were broadcasted by the BS send *naks* to the BS, as already introduced in the previous subsection. The BS receives a corresponding *NAK* and will transmit the next symbol (packet) from the code word. Note, for $t<k$, none of the user terminals will be able to receive k packets, as only $t<k$ packets were broadcasted.

In order to calculate the throughput we are interested in the expected value of the number of required packet transmissions to broadcast all k information packets correctly to all users. Let $p_{\mathrm{n},m}(t)$ denote the probability that a single receiver sends a *nak* after it received t packet, i.e. the probability of unsuccessful reception of k packets after t transmission attempts. Hence, $p_{\mathrm{n},m}(t)$ is given by

$$p_{\mathrm{n},m}(t) = \begin{cases} 1 & t < k \\ \sum_{i=0}^{k-1} \binom{t}{i} (1-p_m)^i \, p_m^{t-i} & t \geq k. \end{cases}$$

Let $p_{\mathrm{N}}(t)$ denote the probability of a *NAK* reception at the BS after t broadcast attempts that equals to the probability that at least one user terminal sends a *nak* after t transmissions. Thus, $p_{\mathrm{N}}(t)$ can be written as

$$p_{\mathrm{N}}(t) = 1 - \prod_m \left[1 - p_{\mathrm{n},m}(t)\right] = \begin{cases} 1 & t < k \\ 1 - \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t}{i} (1-p_m)^i \, p_m^{t-i}\right] & t \geq k. \end{cases}$$

Let T be a random variable denoting the number of required overall broadcast attempts (channel uses), including the k systematic transmissions and all redundancy packet transmissions, to broadcast k packet correctly to **all** users. Let $\Pr\{T > t\}$ denote the probability that more than t transmissions are required to deliver all k packet to all receivers correctly, which is given by the probability that after t transmissions a *NAK* is received and, thus there are still user terminals requiring incremental redundancy. Hence, $\Pr\{T > t\}$ is given by

$$\Pr(T > t) = p_{\mathrm{N}}(t) = \begin{cases} 1 & t < k \\ 1 - \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t}{i} (1-p_m)^i \, p_m^{t-i}\right] & t \geq k. \end{cases}.$$

Following from this, the probability $\Pr(T = t)$ that exactly T transmissions are required to transmit k packets correctly to all M users for $t \geq k$ is obtained by

$$\begin{aligned} \Pr(T = t) &= \Pr(T > t-1) - \Pr(T > t) = \\ &= p_{\mathrm{N}}(t-1) - p_{\mathrm{N}}(t) = \\ &= 1 - \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t-1}{i} (1-p_m)^i \, p_m^{t-1-i}\right] - \left[1 - \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t}{i} (1-p_m)^i \, p_m^{t-i}\right]\right] = \\ &= \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t}{i} (1-p_m)^i \, p_m^{t-i}\right] - \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t-1}{i} (1-p_m)^i \, p_m^{t-1-i}\right]. \end{aligned}$$

For $t < k$ it is obvious that $\Pr(T = t) = 0$.

Following from this and assuming n sufficiently large the expected value $E\{T\}$ for the number of required broadcast transmissions can be written as

$$E\{T\} = \sum_{t=1}^{\infty} t \cdot \Pr\{T = t\} = \sum_{t=k}^{\infty} t \cdot \left\{ \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t}{i} (1 - p_m)^i \, p_m^{t-i} \right] - \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t-1}{i} (1 - p_m)^i \, p_m^{t-1-i} \right] \right\},$$

which allows us to give the relative throughput $\eta(\mathbf{p})$ of the p-t-m incremental redundancy retransmission scenario exploiting Reed-Solomon codes as

$$\eta(\mathbf{p}) = \frac{k}{E\{T\}} = \frac{k}{\sum_{t=k}^{\infty} t \cdot \left\{ \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t}{i} (1 - p_m)^i \, p_m^{t-i} \right] - \prod_m \left[1 - \sum_{i=0}^{k-1} \binom{t-1}{i} (1 - p_m)^i \, p_m^{t-1-i} \right] \right\}}.$$

Please note that in practice the available redundancy is limited by n resulting in a residual error rate. However, the residual error rate is that small that it is negligible for our scenario. This will be discussed in more detail in the future, in combination with possible limitation of the number of retransmissions, the frequency of the NAKs, etc.